Complexity reduction by combining Time Reversal and IR-UWB

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***Abstract*—In this paper we deal with the problem of minimizing the complexity of an IR-UWB system by the introduction of Time-Reversal, under a power constraint and fixing a reachable performance in terms of BER. An approximate trade-off in the choice of the number of taps at the transmitter and the number of fingers at the receiver is proposed.**

I. Introduction

In a classical communication chain, the reception of a signal facing multipath with a bank of single user matched filters (SUMF) is accomplished by the RAKE receiver, whose number of fingers increases with the number of paths of the multipath channel.

We may introduce a FIR filter at the transmitter (TX) in order to decrease the number of fingers at the receiver (RX), fixing a performance in terms of BER. Time-Reversal (TR) is a technique that chooses a scaled version of the channel impulse response (CIR) reversed-in-time as the impulse response of this filter, called Time-Reversal prefilter or precoder.

In this perspective, we call complexity of the system the total number of taps in TR and fingers in RAKE.

An advantage of TR is that it can be used to switch the complexity from RX to TX, allowing to remain with a 1-finger RAKE (1-RAKE). However, it can be combined with an all-RAKE to improve the performance of the system by exploiting the property of TR to increase the energy that can be collected by RX.

From a different point of view, in order to distribute the complexity between TX and RX, partial TR and RAKE are feasible by suitable sub-selection of taps and fingers.

The trade-off between the number of taps and fingers has been addressed in [6] with the assumption of an equal amount of energy at the receiver side. On the contrary, in this work we assume the more usual power constraint at TX, that in turn allows to isolate the contribution of TR, from that of transmitted power, to the performance of the system.

In this work we focus on MUI-free channel. A more realistic analysis, motivated by the nature of UWB systems as underlay systems [7] [8], should consider the MUI effect on the trade-off.

In this paper we want to study how to minimize the complexity of a IR-UWB system, fixing a performance in terms of BER, by the use of Time Reversal.

II. Description of the model

A. Signal model

The UWB communication system we consider (see Figure 1) adopts an Impulse-Radio signaling scheme [5], meaning that the ultrawide bandwidth characteristic is obtained radiating a (train of) basic pulse waveform $g(t)$ of very short duration, with a compact support in the *chip interval* $[0, T_\mathrm{C}]$. We focus on binary signaling schemes, both *orthogonal* and *antipodal*, and in particular PPM and PAM respectively. Wireless access with many transmitters and receivers in the network is provided by a time-hopping code (inherently periodic, of N_P say), uniformly distributed in $\mathcal{U}[0, N_\mathrm{H}] \cap \mathbf{Z}$, that delays $g(t)$ in one of the N_H chips composing a *frame* ($T_\mathrm{S} = N_\mathrm{H} T_\mathrm{C}$). Thus, the transmitter has a (fixed) vector $\mathbf{c} = [c_0, \ldots, c_{N_\mathrm{P}-1}]^T$ of discrete i.i.d. uniform random variables known by the receiver. For notational convenience, in the following we will use c_i instead of $c_{i (\mathrm{mod}\ N_\mathrm{P})}$. Furthermore, in order to introduce redundancy, the modulator has the ability of coding a bit of information into N_S symbols, e.g. with a *repetition code* (in that case, $T_b = N_\mathrm{S} T_\mathrm{S}$).

The transmitted signal can be written as follows

$$s(t) = \sqrt{\mathcal{E}_b} \sum_{n \geq 0} g(t - nT_b; b_n)$$

where

$$g(t; b_n) = \begin{cases} \displaystyle\sum_{i=0}^{N_\mathrm{S}-1} (-1 + 2b_n) g(t - iT_\mathrm{S} - c_{nN_\mathrm{s}+i} T_\mathrm{C}) & \text{for PAM,} \\ \displaystyle\sum_{i=0}^{N_\mathrm{S}-1} g(t - iT_\mathrm{S} - c_{nN_\mathrm{s}+i} T_\mathrm{C} - b_n \varepsilon) & \text{for PPM.} \end{cases}$$

Hereinafter, we will consider $N_\mathrm{S} = 1$. Furthermore, adopting a block transmission paradigm, w.l.o.g. we can rewrite the

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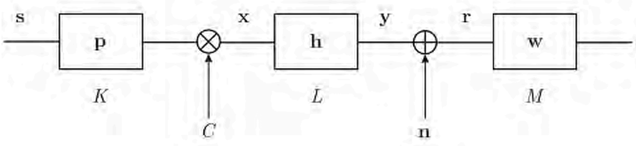

FIG. 1: Basic model. (K, L, M) denotes that the precoder has K taps, the channel has L paths and the RAKE has M fingers.

previous waveforms for the first information bit only:

$$g(t; b) = \begin{cases} (-1 + 2b)g(t - cT_{\mathrm{C}}) & \text{for PAM} \ , \\ g(t - cT_{\mathrm{C}} - b\varepsilon) & \text{for PPM} \ . \end{cases}$$

Thus, regardless of the time-hopping shift, in the PAM case, the basic pulse is simply $g(t)$. Because of the dimensionality of the signal space (that is 1), the two possible signals to be transmitted are:

$$s_m(t) = (-1 + 2m)g(t) \ , \quad m = 0, 1 \ ,$$

and a base for this space is (for instance) given by $\mathcal{B} = \{s_1(t)\}$.

In the PPM case, the signals are:

$$s_m(t) = g(t - m\varepsilon), \quad m = 0, 1 \ .$$

If $\varepsilon \geq T_{\mathrm{M}}$, they are orthogonal, being T_{M} the duration of the pulse. The signal space has dimension 2 and a base is (for instance) given by $\mathcal{B} = \{s_0(t), s_1(t)\}$.

B. Channel

The channel statistic for UWB communication is unique due to the ultra high resolution of receivers. Both IEEE 802.15.3a [3] (used in simulations) and IEEE 802.15.4a channel models [2] are based on the seminal work of Saleh and Valenzuela [4]. The simplest way to describe the channel is the following:

$$h(t) = \sum_{\ell=1}^{L} \alpha_\ell \delta(t - \tau_\ell) \ .$$

Note 1. *We stress that* L *is the number of paths of the channel.*

C. Precoder

We apply here the time-reversal concept introducing a filter that is nothing but the channel reversed (= inverted) in time:

$$p(t) = \sum_{k=1}^{L} \alpha_k \delta(t + \tau_k).$$

We do not care about the causality of this filter, but it is evident that in a real experiment it would be necessary a delay (at least) equals to τ_L. In general, we could use a lesser complex filter with $K \leq L$ taps, selecting only the K strongest paths of $h(t)$. In this case we have:

$$p(t) = \sum_{k \in \mathcal{K}} \alpha_k \delta(t + \tau_k) \ ,$$

where $\mathcal{K} \subseteq \{1, 2, \ldots, L\}$, $|\mathcal{K}| = K$.

Note 2. *We stress that* K *is the number of taps of the precoder.*

Note 3. To carry out a correct comparision of performance among various systems, we introduce a power constraint for the transmitter: we assume that the power of the sent signal is constant. We take into account this as follows. Let be $x(t)$ the signal sent, thus (see FIGURE 1)

$$x(t) = C(s * p)(t) = C \sum_{k \in \mathcal{K}} \alpha_k \, s(t + \tau_k) \ , \quad C \in \mathbf{R}^+ \ .$$

Assumption 1. We assume that $g(t)$ has a support $[0, T_{\mathrm{M}}]$ with $0 < T_{\mathrm{M}} \leq \min_{i \neq j} |\tau_i - \tau_j|, \ 0 \leq i, j \leq L$.

With this assumption, the power constraint reads as $\mathcal{E}_x = \mathcal{E}_s$, and therefore

$$\mathcal{E}_x = C^2 \mathcal{E}_s \sum_{k \in \mathcal{K}} \alpha_k^2 = C^2 \mathcal{E}_s \|\mathbf{p}\|^2 \implies C = 1/\|\mathbf{p}\| \ .$$

D. RAKE

The optimum demodulator for processing a wideband signal is known as RAKE correlator [1]. It was introduced by Price and Green in 1958 and it is the filter matched to the whole *useful* signal (that is without neither noise nor interference) at the receiver. In our framework, $y(t) = (x * h)(t)$ is the useful signal and $r(t) = y(t) + n(t)$ is the received signal corrupted by the WGN $n(t)$ with variance σ_n^2. Then, the RAKE is implemented by $w(t) := y(-t)$, whose output is aptly sampled. Actually, we can also use a selective RAKE with M fingers that choose the M strongest path of the equivalent channel $h_{\mathrm{e}}(t) = C(h * p)(t)$,

$$h_{\mathrm{e}}(t) = \frac{1}{\|\mathbf{p}\|} \sum_{k \in \mathcal{K}} \sum_{\ell=1}^{L} \alpha_k \alpha_\ell \delta(t - \tau_\ell + \tau_k).$$

Note 4. Hereinafter, we fix L. Then we always have the following bounds: $K \leq L$ and $M \leq 1 + K(K-1) + K(L-K) = 1 + K(L-1)$.

E. Equivalence: $(1, K) \sim (K, 1)$

We write $(K, M) \sim (K', M')$ to denote two systems that have the same performance in terms of BEP. This is an equivalence relation (symmetric, reflexive and transitive), thus it partitions $\mathbf{R}_+^2$ into equivalence classes (of systems with the same performance).

In this section we prove that $(K, L, 1) \sim (1, L, K)$, $1 \leq K \leq L$, thus a *partial*-TR with a 1-RAKE has the same performance of a *partial*-RAKE without TR, provided that they have the same number of taps.

The signal received (as illustrated in FIGURE 1) is

$$r_m(t) = \|\mathbf{p}\| s_m(t) + \frac{1}{\|\mathbf{p}\|} \sum_{k \in \mathcal{K}} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{L} \alpha_k \alpha_\ell s_m(t - \tau_\ell + \tau_k) + n(t) \ ,$$

where $s_m(t)$ is the signal that modulates a bit m. For the sake of simplicity, let us continue with PAM analysis only. A 1-RAKE will correlate this signal with the highest path, i.e. the correlation metric [1] will be (we drop the explicit reference to the time-hopping code)

$$CM_1 = \langle r_1(t), \|\mathbf{p}\| s_1(t) \rangle =: A + \nu$$

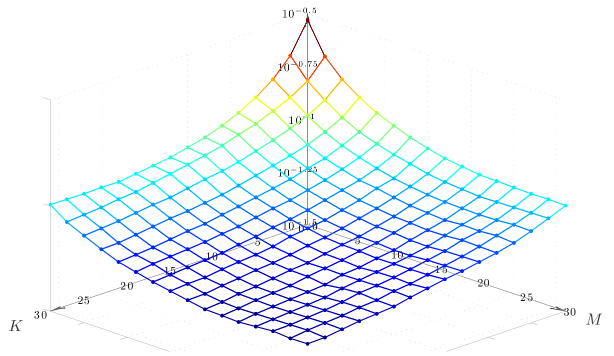

FIG. 2: BER as a function of K and M with fixed $\mathcal{E}_b/N_0 = 4$ [dB].

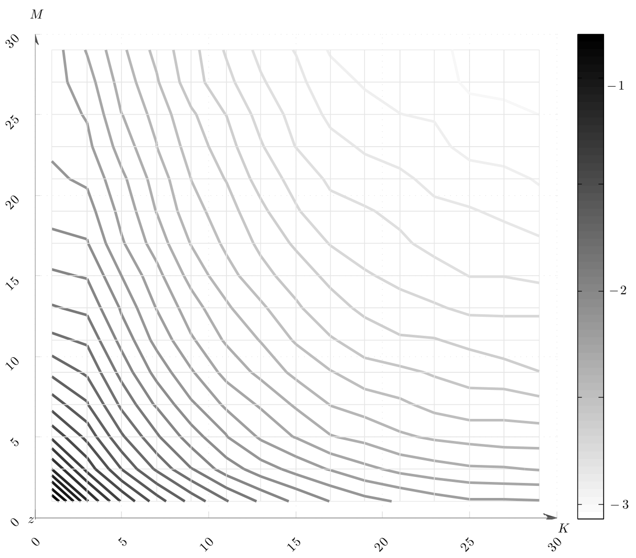

FIG. 3: Iso-BER curves with fixed $\mathcal{E}_b/N_0 = 8$ [dB].

where $A := \mathcal{E}_s\|\mathbf{p}\|^2$ and

$$\nu := \langle n(t), \|\mathbf{p}\| s_1(t)\rangle \sim \mathcal{N}\left(0, \sigma_n^2 \mathcal{E}_s \|\mathbf{p}\|^2\right) \ .$$

It yields

$$\gamma_b := \frac{A^2}{2\sigma_\nu^2} = \frac{\mathcal{E}_s\|\mathbf{p}\|^2}{N_0} \ , \qquad \sigma_n^2 := N_0/2 \ ,$$

that is the same well-known result of a selective K-RAKE.

Remark 1. This result shows the remarkable property that, having fixed L, in the plane (K, M), $K, M \geq 1$, an iso-BEP curve that starts in $(k, 1)$ ends in $(1, k)$. It is simple to show that this is valid irrespective of the modulation type (PPM or PAM).

Note 5. Both the systems collect the same energy $\mathcal{E} = \mathcal{E}_s \sum_{k=1}^{K} \alpha_k^2$.

III. MINIMIZING THE TOTAL NUMBER OF TAPS AND FINGERS

In FIGURE 2 and 3, as already stated via theoretical computations, we can see that the generic curve in the plane (K, M) that starts in $(k, 1)$ ends in $(1, k)$. This suggests a rule-of-thumb that provides a fairly good fitting considering hyperbolas as these curves.

PARAMETER	VALUE	UNITS
R (ray arrival rate)	2	[GHz]
C (cluster arrival rate)	20	[MHz]
s_0 (ray decay factor)	2	[ns]
τ_0 (cluster decay factor)	5	[ns]
σ_1 (cluster fading std. dev.)	3.3941	[dB]
σ_2 (ray fading std. dev.)	3.3941	[dB]

TABLE I: Channel model parameters.

Fixing a (reachable) BEP, we can start choosing the minimum number c of fingers, in a system with a RAKE receiver and without TR, that allows to attain the desired BEP. The system $(1, c)$ is the simplest that guarantees an average BER lesser or equal to the desired one. Then we move on the curve $(k, c/k)$, shifting the complexity from the receiver to the transmitter. We can switch all the complexity, arriving at $(c, 1)$, or just a part of it.

Now we want to minimize the total number of taps and fingers, fixing a performance. In order to formalize the problem, let be $(k, m) \in \mathbf{Z}_+^2$ the pair denoting the number of taps and fingers employed, respectively. Thus we want to solve the problem

$$\begin{cases} \min & k + m \qquad\qquad (k, m) \in \mathbf{Z}_+^2 \ , \\ \text{s.t.} & km = c \ , \end{cases}$$

where c is a feasible constant that depends on the performance to reach. We may generalise this problem assigning a cost to each choice. In this case the problem becomes

$$\begin{cases} \min & ak + bm \qquad\qquad (k, m) \in \mathbf{Z}_+^2, \quad a, b \in \mathbf{R}_+ \ , \\ \text{s.t.} & km = c \end{cases} \ .$$

We will proceed embedding the problem in $\mathbf{R}_+^2$ and then choosing the nearest integer pair in the lattice $\mathbf{Z}_+^2$, altough of course this couldn't be the integer solution. By elementary calculus, we find that $k^\star = \sqrt{bc/a}$, $m^\star = \sqrt{ac/b}$ and the attained minimum is $2\sqrt{abc}$. If $a = b = 1$, the minimum number of taps as well as fingers is $\sqrt{c}$.

This result suffers of some inaccuracy due to the extremely simple model adopted accepting the hyperbola hypothesis. Let us analyse the region of validity of this approach.

A. R_1. Small values of c, $\{c \ll L\}$.

The major limitation of this approach is evident where we consider a system $(1, L, c)$ with $c \ll L$ (see FIGURE 4) because the hyberbola under-estimates the number of taps that we need. In this case the best fitting is actually linear and leads to the equivalence $(1, L, c) \sim (1 + r, L, c - r)$, meaning that the complexity can be shifted proportionally from the receiver to the transmitter. The most fair condition, provided that transmitter and receiver have identical costs ($a = b$), is a system $(c/2, L, c/2)$ that splits the complexity in equal parts.

The linear approximation is valid for $c \ll L$ and for higher values of c it is conservative. On the contrary, the hyperbolic approximation is not conservative for $c \ll L$ and leads to a loss in $\mathcal{E}_b/N_0$ of about 2 [dB].

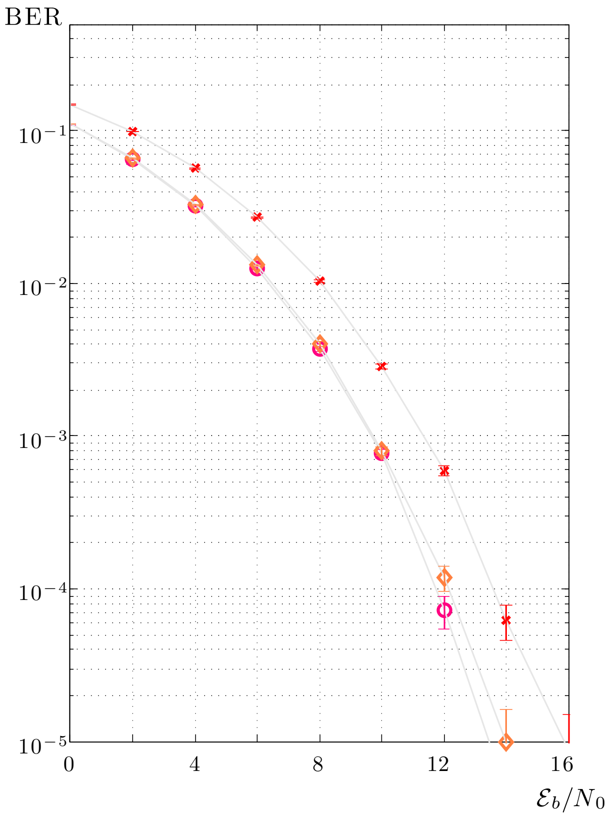

FIG. 4: BER (circle: $K = 16$, $M = 1$; cross: $K = 4$, $M = 4$; diamond: $K = 7$, $M = 7$).

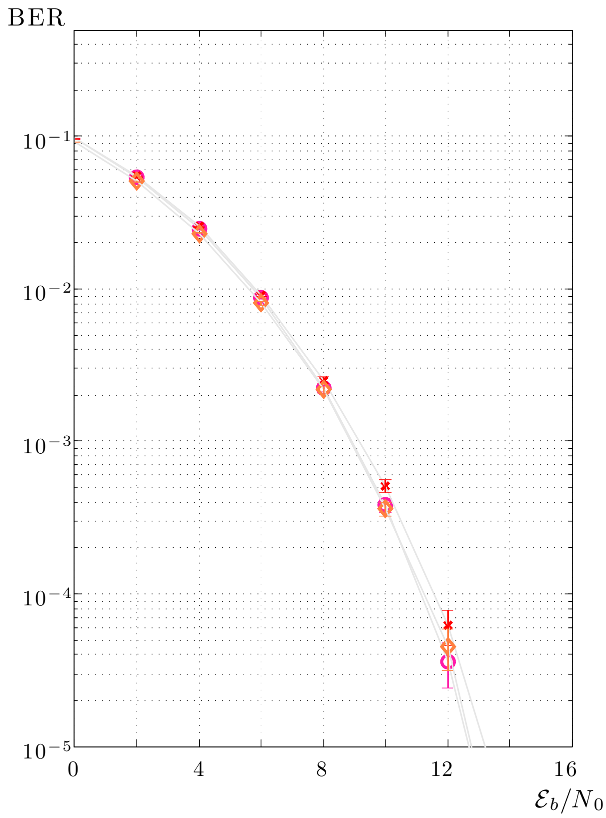

FIG. 5: BER (circle: $K = 50$, $M = 1$; cross: $K = 7$, $M = 7$; diamond: $K = 10$, $M = 10$).

B. R_2. Other values of c, $R_2 := \{1 \leq c \leq L\} - R_1$.

The greater the number of initial paths, the better the approximation $(1, L, c) \sim (\sqrt{c}, L, \sqrt{c})$. In FIGURE 5 we show that the loss in this case is negligible (< 0.5 [dB]). In any case, it is evident from simulations that it may be sufficient to consider $\sqrt{2c}$ instead of $\sqrt{c}$ to achieve a zero loss.

IV. CONCLUSION

We have studied the complexity trade-off between TR precoder and RAKE receiver in IR-UWB systems in AWGN MUI-free scenarios.

We have found via simulations an upper-bound and a lower-bound to the complexity, being very tight when few and many paths are considered, respectively.

Future work will address the effect of the MUI on the found trade-off as well as stronger analytical results.

V. ACKNOWLEDGMENT

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