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Application of Hybrid Models to the Design of Ultra Wide Band Self-organizing Networks

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***Abstract*— The problem is modeling the operation of a self-organizing network of nodes that operate according to the UWB principle. We propose hybrid systems as a powerful framework to model the rules that lead to the formation of the network and in particular an admission control procedure that is capable to handle both continuous and discrete perturbations. Nodes may adjust their rules of operation based on the perception of the environment by an elected node, serving as the observer that is aware of context, evaluates, and selects one strategy of operation.**

***Keywords*— Hybrid systems, Self-organizing networks, Ultra Wide Band**

I. INTRODUCTION

Wireless networks are capturing increasing attention in the industrial and scientific community as wireless technology matures and the number of devices communicating wirelessly is growing exponentially. In a wireless self-organizing network, nodes are forced to adapt the rules of operation according to changes and perturbations that are in general asynchronous with respect to node dynamics. In addition, nodes must cope with noise and other unpredictable events, such as atmospheric changes or mobility random patterns, which introduce additional uncertainty in node behavior. Hence, an accurate modeling of a node in a wireless network requires a mathematical framework where continuous and discrete dynamics are appropriately defined.

In this paper, we formalize a model for a self-organizing network of nodes that operate according to the Ultra Wide Band (UWB) principle [2] using *hybrid systems* (see e.g. [1]); hybrid systems offer in fact the analytical framework for modeling complex systems where continuous dynamics and discrete processes tightly interact.

Given the ultra wide bandwidth of radiated signals, radio devices communicating under UWB rules must operate in severely interfered environments and must control their behavior not to disturb the other nodes of the network. In the proposed model, we adopt cognitive mechanisms [3], [4] in the analysis process that is used by the nodes for determining whether changes in the global network state are appropriate. Network nodes are thus provided with the ability of adapting their rule of operation based on the analysis of the environment in which they operate. The cognitive principle is also integrated in the rules of interaction between nodes. Based on the assumptions above, the set of wireless nodes is considered in our model as a social network that is modeled and analyzed as one single entity.

The issue addressed in this paper is the design of the rules that lead to the formation of the network and, in particular, the design of an admission control procedure that is capable of handling both continuous and discrete perturbations. The network is maintained in a condition of stability by construction, in the sense that there are no oscillations in the network behavior due to changes in the constraints.

This paper is organized as follows. Using the hybrid system formalism, we characterize in Section II the network dynamics as a discrete finite-state automaton where, for each state, specific rules of operation govern the evolution of the network itself. Cognition is introduced in the model by allowing the nodes to adjust their rules of operation based on the perception of the environment by an elected node (the Conscious Node, or CNode), serving as the observer, that is aware of context, evaluates, and selects one strategy of operation. In Section III, we present some concluding remarks highlighting open problems that may be formally stated and analyzed using the proposed hybrid model.

II. ADMISSION CONTROL BY HYBRID MODELING

In the proposed model of a self-organized network, each discrete state of the automaton corresponds to an operation mode, described by the waveform used for pulse shaping. In addition, one particular state of the automaton corresponds to the admission control mode, where the CNode evaluates the possibility of admitting a new node in the network. A transition to this state takes place when a new node is asking for admission in the network and, for simplicity, we assume that the control procedure requires a negligible time to be performed. The automaton is represented in Figure 1 for the very simple case of two waveforms, namely w_1 and w_2.

A continuous variable $N(t)$, the current number of active nodes that are allowed to transmit data over the wireless channel, is associated to each discrete state of the

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automaton. This variable is reset to a new value whenever a transition occurs, as described in the next subsection.

In each state, the system receives different inputs ranging from Radio Frequencies (RF) stimuli from the environment in agreement with the model proposed by [3], to indicators of the attenuation that is present over the active links. These attenuation indicators are used by the active nodes for evaluating at time t both potential transmission parameters as well as their capability to comply with the transmission constraints that are communicated by the CNode through a time dependent set of parameters named $\mathbf{K}(t)$.

The network interacts with the environment through the CNode. We suppose that changes in the environment are related to events such as the arrival of an interferer or the creation of a coexisting network, and as such the temporal scale of such perturbations is sensibly longer with respect to other events such as packet generation. The CNode sends, during connection, continuous updates regarding transmission parameters. In turn, these parameters are used by the active node for adjusting its communication.

This time-varying set of parameters $\mathbf{K}(t)$, is formed as follows:

1) the waveform w^* that must be used for pulse shaping. Different pulse shapes can be selected for transmitting data over the wireless channel; w^* is the one that better adapts with the environment, as well as with thermal noise and Multi User Interference (MUI) patterns;
2) the power level $P_m(w^*)$ that is required at the CNode in order to comply with the requirement of a given signal to noise ratio threshold; power P_j that node j must use for transmission depends on the power attenuation A_j characterizing the link between node j and the CNode, and can be expressed as
$$P_j = P_m(w^*)A_j, \qquad j = 1, ..., N$$
3) the noise level $\eta_p(w^*)$ that is currently measured at the CNode;
4) the MUI weight $\sigma_m^2(w^*)$ that is associated by the CNode to the pulse shape w^*, corresponding to a measure of the effect of the presence of MUI;
5) the number of active nodes N.

Within the above set, one can identify in the first two parameters w^* and $P_m(w^*)$, constraints that are imposed to the nodes in the network. The noise level $\eta_p(w^*)$ can be interpreted as a continuous disturbance acting on the overall system dynamics. The MUI weight $\sigma_m^2(w^*)$ and the number of active nodes N are information characterizing the current system state.

The time dependent set of parameters $\mathbf{K}(t)$ is evaluated at the CNode using the Received Power Function for N active nodes, according to principles that are peculiar to UWB systems (see [5]). We suppose that the signal containing the above information is sent in broadcast by the CNode at a fixed power level that is pre-determined and known by all nodes.

Each active node j receives the signal conveying $\mathbf{K}(t)$ and, on the basis of received power level, can estimate the attenuation A_j characterizing its path to the CNode. Node j determines both power and rate to be used in its future transmissions to the CNode according to a procedure presented in [5]. We assume here that the variations of the environment, which are reflected in the time-varying set $\mathbf{K}(t)$, are tolerable by all nodes in the network.

In the next subsection, we formally describe the network dynamics using the hybrid systems formalism.

A. The hybrid model

A hybrid model $\mathcal{H}$ for the network can be defined by introducing the tuple $\mathcal{H} = (\Xi, \mathbf{\Sigma}, S, E, R)$:

- $\Xi = \mathbf{Q} \times \mathbb{R}$ is the hybrid state space; $\mathbf{Q} = \mathbf{W} \cup \{\widehat{q}\}$, $\mathbf{W}$ is the finite set of waveforms that can be used for pulse shaping; the state $\widehat{q}$ represents the fact that a candidate node is waiting for admission in the network;
- $\mathbf{\Sigma}$ is the set of discrete inputs; $\mathbf{\Sigma} = \mathbf{\Sigma}^c \cup \mathbf{\Sigma}^d \cup \{\widehat{\sigma}\}$, where $\mathbf{\Sigma}^c$ is the set of discrete controls and $\mathbf{\Sigma}^d$ is the set of discrete disturbances and
$$\begin{aligned} \mathbf{\Sigma}^c &= \mathbf{\Sigma}^c_{\mathbf{W}} \cup \mathbf{\Sigma}^c_a \\ \mathbf{\Sigma}^c_{\mathbf{W}} &= \{\sigma_{ij}, i, j \in J\} \\ \mathbf{\Sigma}^c_a &= \{OK, NO\} \\ \mathbf{\Sigma}^d &= \{\sigma_a, \sigma_l\} \end{aligned}$$
The discrete controls σ_{ij} model the decision taken by the CNode to commute from pulse shape w_i to pulse shape w_j. The discrete controls $\{OK, NO\}$ correspond, respectively, to the decision taken by the CNode to accept or not to accept a candidate node in the network. The discrete disturbances σ_a and σ_l represent, respectively, the request by some candidate node to enter the network and the event that a node leaves the network. Finally, the discrete input $\widehat{\sigma}$ is an endogenous signal that is generated when changes in the environment and in radio propagation are no more compliant with node's requirements.
- S is a map associating to every discrete state in $\mathbf{Q}$ a dynamical system.
- If the discrete state is $w_i \in \mathbf{W}$, then the dynamical system $S(w_i)$ is described by the equations:
$$\dot{N}(t) = 0$$
$$\mathbf{K}_i(t) = \begin{pmatrix} w_i \\ p_i(t) \\ \eta_p(w_i) \\ \sigma_m^2(w_i) \\ N(t) \end{pmatrix}$$
where
$$p_i(t) = P_m(w_i, t)$$
$$P_m(w_i, t) = \frac{\eta_p(w_i)}{T_S}\left(\frac{1}{SNR_0} - \frac{\sigma_m^2(w_i)}{T_S}(N(t) - 1)\right)$$

$N(t)$ is the continuous state at time t, i.e. the number of current nodes in the network; $\mathbf{K}_i(t)$ is the output, where $p_i(t)$ is the power that the CNode must receive from the $N(t)$ active nodes in order to comply with the requirements of a threshold SNR_0, when the pulse shape is w_i, and T_S is the pulse repetition period. $\eta_p(w_i)$ and $\sigma_m^2(w_i)$, represent the discrete state dependent disturbances. The initial value for the continuous state is $N(0) = 1$ (at the beginning, the CNnode is the only node in the network). The value of the state is reset whenever a transition occurs.

- If the discrete state is $\widehat{q}$, then $S(\widehat{q})$ is described by the equations

$$\dot{N}(t) = 0$$

$$\widehat{\mathbf{K}}(t) = \begin{pmatrix} w^* = \arg\min_{w \in \mathbf{W}} \widehat{P}_{\min}(w) \\ \widehat{p}(t) \\ \eta_p(w^*) \\ \sigma_m^2(w^*) \\ N(t)+1 \end{pmatrix}$$

where

$$\widehat{p}(t) = \widehat{P}_m(w^*, t)$$

$$\widehat{P}_m(w^*, t) = \frac{\eta_p(w^*)}{T_S}\left(\frac{1}{SNR_0} - \frac{\sigma_m^2(w^*)}{T_S} N(t)\right)$$

$N(t)$ is the continuous state at time t, $\widehat{\mathbf{K}}(t)$ is the output produced by the CNnode at time t. The state $\widehat{q}$ corresponds to the control admission mode and the role of the output $\widehat{\mathbf{K}}(t)$ is therefore discussed in Subsection 2.2 describing the admission control algorithm.

- $E \subset \mathbf{Q} \times \mathbf{\Sigma} \times \mathbf{Q}$ is a collection of transitions.

$$E = E^c \cup E^d \cup E^{inv}$$

where

$$\begin{aligned} E^c &= E_{\mathbf{W}}^c \cup E_a^c,\ E^d = E_a^d \cup E_l^d \\ E_{\mathbf{W}}^c &= \{(w_i, \sigma_{ij}, w_j), \sigma_{ij} \in \mathbf{\Sigma}_{\mathbf{W}}^c, w \in \mathbf{W}\} \\ E_a^c &= \{(\widehat{q}, \sigma, w), \sigma \in \mathbf{\Sigma}_a^c, w \in \mathbf{W}\} \\ E_a^d &= \{(w, \sigma_a, \widehat{q}), w \in \mathbf{W}\} \\ E_l^d &= \{(w, \sigma_l, w), w \in \mathbf{W}\} \\ E^{inv} &= \{(w, \widehat{\sigma}, w), w \in \mathbf{W}\} \end{aligned}$$

- The transitions in E^c are controlled (in Fig. 1 these transitions are represented by solid arrows).

- A transition (w_i, σ_{ij}, w_j) in $E_{\mathbf{W}}^c$ models the decision, taken at some time t by the CNode, of commuting from pulse shape w_i to pulse shape w_j, for transmitting data over the wireless channel. This transition takes place when w_j is the pulse shape that, at time t, better adapts to the time-varying environment, as well as thermal noise and MUI patterns, or when $p_i(t) > P_m(w_j, t)$. In the latter case, w_j is such that $P_m(w_j, t) \leq P_m(w_h, t)$, $\forall w_h \in \mathbf{W}$.

- The transitions in E_a^c occur when the CNode decides to accept or not to accept a candidate node in the network.

- The transitions $\{(w, \sigma_a, \widehat{q}), w \in \mathbf{W}\}$ are not controlled (switching transitions) and represent the request of entering the network by some candidate node.

- The transitions $\{(w, \sigma_l, w), w \in \mathbf{W}\}$ (dashed arrows in Fig. 1) are not controlled and represent the fact that a node could leave the network because its activity is terminated for reasons that range from no more data packets to transmit, to node failure, to power exhaustion.

- The transitions in E^{inv} (dot-line arrows in Fig. 1) occur because changes in the environment (as sensed by the CNode) and in radio propagation (as perceived by the active nodes) are no more compliant with node's requirements. Then, the node leaves the network).

For simplicity, we assume that simultaneous transitions are not allowed.

- $R : \Xi \times E \rightarrow \Xi$ and

$R((q_i, x), e) = (q_h, x)$,	$e = (q_i, \sigma, q_h) \in E_{\mathbf{W}}^c$
$R((q_i, x), e) = (q_h, x+1)$,	$e = (q_i, OK, q_h) \in E_a^c$
$R((q_i, x), e) = (q_h, x)$,	$e = (q_i, NO, q_h) \in E_a^c$
$R((q_i, x), e) = (\widehat{q}, x)$,	$e = (q_i, \sigma, \widehat{q}) \in E_a^d$
$R((q_i, x), e) = (q_i, x-1)$,	$e = (q_i, \sigma, q_i) \in E_l^d$
$R((q_i, x), e) = (q_i, x-1)$,	$e = (q_i, \sigma, q_i) \in E^{inv}$

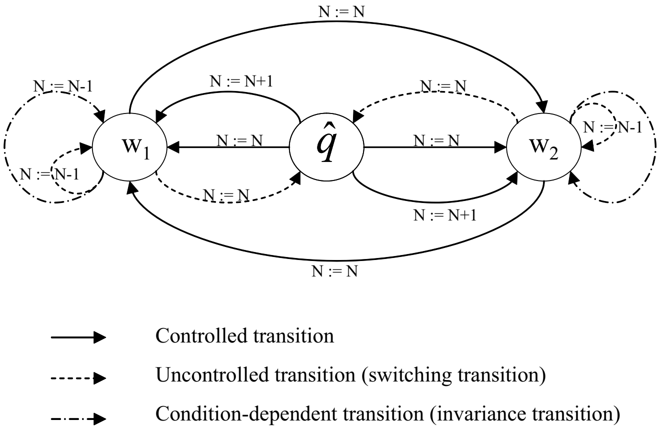

Fig. 1. Hybrid model

B. The control algorithm

The main control objective is to maximize the number of active nodes in the network, while preserving transmission requirements. In fact, when a node asks for admission, i.e. when the current discrete state at time t is $\widehat{q}$, the CNode evaluates the possibility of admitting the new element in the network, by computing a hypothetical set of parameters $\widehat{\mathbf{K}}(t)$. The use of this information is twofold. First, it serves to the current active nodes in order to check whether constraints for transition are compatible with their specifications and inform the CNode. Willingness to transition of all nodes is a necessary condition for transition. Second, the information in $\widehat{\mathbf{K}}(t)$ is used by candidate node for evaluating its willingness to join the network. A candidate node that listens to $\widehat{\mathbf{K}}(t)$ must agree in accepting those constraints for the transition to take place. The two conditions above correspond to guard conditions that must be satisfied in order for the transition $(\widehat{q}, OK, w^*) \in E_a^c$ to take place (where w^* is the first component of $\widehat{\mathbf{K}}(t)$). If the above conditions are not fulfilled, then a transition $(\widehat{q}, NO, w_h)$ in the set E_a^c takes place, where $w_h = \arg\min_{w \in \mathbf{W}} P_m(w)$. Therefore, a new node is admitted only if none of the current active nodes is forced to leave the network as a result of the new admission.

At each time, the network is controlled so that the power level is minimum with respect to the number of active nodes, possible choices of pulse shaping and environmental parameters. As a consequence, the described control strategy minimizes the energy consumption in the network, which is a beneficial effect in wireless communication.

III. CONCLUSIONS AND FUTURE WORK

Using the hybrid systems formalism, we characterized self-organizing network dynamics as a finite state automaton where, for each discrete state, specific rules of operation govern the evolution of the network itself.

Several benefits are obtained by introducing the hybrid system model for the design of UWB self-organizing networks:

- The formal description resulting from the adoption of the hybrid system formalism allows a better understanding of some important properties of the system. As an example, it is possible to characterize the trade off that exists between the complexity of a real-time and precise scanning of the external environment vs. the improvement in system efficiency that is achieved when the nodes can rapidly adapt themselves to the varying condition of the operating scenario. Based on this trade-off, we could investigate the existence of sub-optimal but computational efficient strategies, where the capability of the nodes to adapt to the external environment is limited and depends upon the current state of the automaton.
- Using the hybrid system model, it is possible to optimize the distribution of functional specifications among the different components of the system. For example, we can analyze how system performance is affected whenever some of the functionalities that are associated to active nodes are associated to the CNode and vice-versa.
- The hybrid formalism may help to predict in which states the automaton will spend most of the time, or the maximum number of nodes of the network. This information is of fundamental importance for network designers.
- The characterization of the wireless network as a hybrid system facilitates the analysis of the stability [6] of the overall system. This task is by no means trivial since we assume that the nodes dynamically adapt transmission parameters and rules of operation to external stimuli.

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