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A Collision-Based Model for Multi User Interference in Impulse Radio UWB Networks

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***Abstract* – Modeling Multi User Interference (MUI) in Impulse Radio (IR) – Ultra Wide Band (UWB) networks is addressed in this paper. The reference scenario consists of multiple asynchronous users transmitting IR-UWB signals using Pulse Position Modulation (PPM) in combination with Time Hopping (TH) coding. We provide a novel analytical expression for the average BER based on the observation that interference in IR is provoked by collisions occurring between pulses belonging to different transmissions. The proposed method requires specification of a similar set of system parameters as Gaussian-based approaches, but shows improved accuracy in estimating BER.**

I. INTRODUCTION

Different methods have been proposed in the recent past for evaluating the effect of Multi User Interference (MUI) on the performance of Impulse Radio-Ultra Wide Band (IR-UWB) networks. Earlier contributions ([1]-[2]) were inspired by the legacy of CDMA reference literature [3] adopting the Standard Gaussian Approximation (SGA) model for MUI. According to this model, the cumulative effect of all disturbing contributions at the receiver is assumed to be an additive Gaussian noise with uniform power spectral density over the range of frequencies of interest. Further investigations showed, however, as in the CDMA case [4], that the SGA provides weak estimations of BER when low values of user bit rate [5] or sparse topologies [6] are considered. Recent papers ([7]-[9]) propose analytical non-Gaussian approaches leading to exact BER expressions at the price of increased computational complexity. These methods do not express, however, BER in an explicit form, and therefore finding a direct relation between system parameters and performance remains an open question. Different perspectives for modeling interference in UWB networks were introduced in [10] and [11]. In [10], the UWB signal is modeled as a filtered Poisson random signal characterized by an average count rate λ (pulse inter-arrival time). It is shown that when λ is large, the interference provoked by an UWB signal tends to a Gaussian random process. In [11], the authors derive an approximation of the probability density function (pdf) for the interference noise at the output of a 2PPM-IR-UWB receiver. Such a pdf, however, is derived for a specific waveform shape, that is the second derivative of the Gaussian pulse, and under the assumptions of power control at the reference receiver, and in the absence of thermal noise. A novel perspective that explicitly takes into account the peculiar way in which information is structured in IR transmissions was introduced in [12]. In [12], MUI was modeled based on the observation that interference in IR is provoked by collisions occurring between pulses belonging to different transmissions. The probability of error over a single bit was in [12] explicitly related to the probability of experiencing pulse overlaps by more than a pre-defined threshold, that is, a pulse error was defined without introducing a receiver structure. The present work extends [12] by redefining the event of pulse error based on a complete receiver structure definition. In particular, the model for the receiver includes soft detection which produces an estimate of a current bit value by collecting information conveyed by the set of pulses representing it. In this sense, the proposed model moves away from the original scheme in which a typical hard detection was implicitly assumed. Note that pulse collision might also turn out to be constructive, that is, collision may result in an increased probability of correct bit detection. This circumstance was taken into account in [12] under the rather simplistic hypothesis that the probability of having an error on the bit given a collision is 0.5. In the present work, we introduce a refined definition for the probability of bit error given that one or more collisions have occurred, which also incorporates the effect of thermal noise in the receiver.

The paper is organized as follows. Section 2 defines the system model. Section 3 introduces the new MUI model. Section 4 presents results obtained by simulation of a network composed of a few nodes. Section 5 contains the conclusions.

II. SYSTEM MODEL

The system model consists of a reference transmitter TX emitting IR-UWB-TH-PPM signals to a reference receiver RX. The binary sequence **b** generated by TX is formed by independent and identically distributed random variables with equally probable bits. The transmitted signal is:

$$s_{TX}(t) = \sqrt{E_{TX}} \sum_j p_0\left(t - jT_S - \theta_j - \varepsilon b_{\lfloor j/N_S \rfloor}\right) \tag{1}$$

where $p_0(t)$ is the energy-normalized waveform of the transmitted pulses, E_{TX} is the energy of each pulse, T_S is the average pulse repetition period, $0 \leq \theta_j < T_S$ is the time shift of the j-th pulse provoked by the TH code, ε is the PPM shift, b_x is the x-th bit of **b**, N_S is the number of pulses transmitted for each bit, and $\lfloor x \rfloor$ is the inferior integer part of x. According to (1), the PPM modulator introduces a delay ε on all N_S pulses corresponding to a "1" bit.

A general flat channel model is assumed. The impulse response for the channel is given by $h(t)=\alpha\delta(t-\tau)$, where α and τ are the amplitude gain and propagation delay. TX and RX are assumed to be perfectly synchronized, that is, RX has perfect knowledge of τ. The channel output is corrupted by thermal noise and MUI generated by N_i interfering IR-UWB devices. The received signal thus writes:

$$s_{RX}(t) = r_u(t) + r_{mui}(t) + n(t) \tag{2}$$

where $r_u(t)$, $r_{mui}(t)$, and $n(t)$ are the useful signal, MUI, and thermal noise, respectively. As regards $r_u(t)$, one has:

$$r_u(t) = \sqrt{E_u} \sum_j p_0\left(t - jT_S - \theta_j - \varepsilon b_{\lfloor j/N_S \rfloor} - \tau\right) \tag{3}$$

where $E_u = \alpha^2 E_{TX}$. As regards $r_{mui}(t)$, we assume that all interfering signals are characterized by same T_S, and thus:

$$r_{mui}(t) = \sum_{n=1}^{N_i} \sqrt{E^{(n)}} \sum_j p_0\left(t - jT_S - \theta_j^{(n)} - \varepsilon b_{\lfloor j/N_S^{(n)} \rfloor} - \tau^{(n)}\right) \tag{4}$$

where $E^{(n)}$ and $\tau^{(n)}$ are received energy per pulse and delay for the n-th interfering user. The relative delay $\Delta\tau^{(n)} = \tau - \tau^{(n)}$ is assumed to be a random variable uniformly distributed between 0 and T_S. The terms $\theta_j^{(n)}$ and $N_S^{(n)}$ in (4) are the time shift of the j-th pulse and the number of pulses per bit for the n-th user, respectively. TH codes are randomly generated and correspond to pseudo noise sequences. $n(t)$ in (2) is Gaussian noise, with double-sided power density $\mathcal{N}_0/2$.

A coherent correlator followed by a ML detector forms RX. Soft decision detection is performed, that is, the signal formed by N_S pulses is considered as a single multi-pulse signal. The received signal is thus cross-correlated with a correlation mask that is matched with the train of pulses representing one bit. The output of the correlator $Z(x)$, for a generic bit b_x, can be thus expressed as follows:

$$Z(x) = \int_{xN_S T_S + \tau}^{(x+1)N_S T_S + \tau} s_{RX}(t) m_x(t - \tau) dt = Z_u + Z_{mui} + Z_n \tag{5}$$

where $m_x(t)$ is the correlation mask for b_x, i.e.:

$$m_x(t) = \sum_{j=xN_S}^{(x+1)N_S} \left(p_0\left(t - jT_S - \theta_j\right) - p_0\left(t - jT_S - \theta_j - \varepsilon\right)\right) \tag{6}$$

Equation (5) indicates that the decision variable $Z(x)$ consists of 3 terms: the signal term Z_u, the MUI contribution Z_{mui}, and the noise contribution Z_n which is Gaussian with zero mean and variance $\sigma_n^2 = N_S \mathcal{N}_0 (1 - R_0(\varepsilon))$. Bit b_x is estimated by comparing $Z(x)$ with a zero-valued threshold according to the following rule: when $Z(x)>0$ ($Z(x)<0$) decision is "0"("1"). For independent and equiprobable transmitted bits, the average BER is thus:

$$\begin{aligned} \text{BER} &= \frac{1}{2} Prob\left(Z(x) < 0 \mid b_x = 0\right) + \frac{1}{2} Prob\left(Z(x) > 0 \mid b_x = 1\right) = \\ &= Prob\left(Z(x) < 0 \mid b_x = 0\right) \end{aligned} \tag{7}$$

III. ESTIMATING THE BER

Under the SGA hypothesis, Z_{mui} and Z_n would be both modeled as Gaussian variables with zero mean and variance σ_{mui}^2 and σ_n^2, respectively. The average BER writes [13]:

$$BER = \frac{1}{2} erfc\left\{ \sqrt{\frac{1}{2}\left(\left(N_S \frac{E_u}{\mathcal{N}_0}\left(1 - R_0(\varepsilon)\right) \right)^{-1} + \left(\frac{N_S T_S \left(1 - R_0(\varepsilon)\right)^2}{\sigma_M^2 \sum_{n=1}^{N_i} \frac{E^{(i)}}{E_u}} \right)^{-1} \right)^{-1}} \right\} \tag{8}$$

where $R_0(t)$ is the autocorrelation function of the pulse waveform $p_0(t)$, and σ_M^2 is given by:

$$\sigma_M^2 = \int_0^{T_S} \left(\int_0^{T_S} p_0(t - \xi)\left(p_0(t) - p_0(t - \varepsilon)\right) dt \right)^2 d\xi \tag{9}$$

Note that the SGA derives from the central limit theorem and is thus only valid asymptotically.

We now introduce our proposed model which moves away from the SGA approach. We start by observing that the Z_u term in (5) is given by:

$$Z_u = \begin{cases} +N_S \sqrt{E_u}\left(1 - R_0(\varepsilon)\right) & \text{if } b_x = 0 \\ -N_S \sqrt{E_u}\left(1 - R_0(\varepsilon)\right) & \text{if } b_x = 1 \end{cases} \tag{10}$$

according to which (7) can be rewritten as follows:

$$\text{BER} = Prob\left(Z_{mui} < -\left(N_S \sqrt{E_u}\left(1 - R_0(\varepsilon)\right) + Z_n\right)\right) = Prob\left(Z_{mui} < -y\right) \tag{11}$$

where $y = N_S \sqrt{E_u}(1 - R_0(\varepsilon)) + Z_n$ is Gaussian with mean $N_S \sqrt{E_u}(1 - R_0(\varepsilon))$ and variance σ_n^2. We have thus:

$$\text{BER} = \int_{-\infty}^{+\infty} Prob\left(Z_{mui} < -y \mid y\right) p_Y(y) dy \tag{12}$$

where $p_Y(y)$ is the Gaussian probability density function of y.

In our approach, $Prob(Z_{mui} < -y|y)$ takes into account collisions between pulses of different transmissions. The number of interfering pulses N_C is confined between 0 and $N_S N_i$ given N_S pulses per bit, and N_i interfering users. For a given N_i, we can thus write:

$$\begin{aligned} Prob\left(Z_{mui} < -y \mid y\right) &= P_{Z_{mui}}(y,0) P_{CP}(0) + P_{Z_{mui}}(y,1) P_{CP}(1) + \\ &+ \cdots + P_{Z_{mui}}\left(y, N_i N_S\right) P_{CP}\left(N_i N_S\right) \end{aligned} \tag{13}$$

where we have introduced $P_{Zmui}(y,i) = Prob(Z_{mui} < -y|y, N_C = i)$ and $P_{CP}(i) = Prob(N_C = i)$. One obtains:

$$BER = \sum_{i=0}^{N_S N_i} P_{CP}(i) \int_{-\infty}^{+\infty} P_{Z_{mui}}(y,i)\, p_Y(y)\, dy \tag{14}$$

For independent interferers, $P_{CP}(i)$ is:

$$P_{CP}(i) = \binom{N_i N_S}{i} P_{C0}{}^{i} (1 - P_{C0})^{N_i N_S - i} \tag{15}$$

where P_{C0} is the probability that a single interfering device produces a colliding pulse within T_S. P_{C0} can be computed as a fraction of T_S during which the receiver may be affected by the presence of an interfering pulse and produce non-zero contributions to Z_{mui}, and can thus be expressed as follows:

$$P_{C0} = \frac{min(2T_M + \varepsilon, 4T_M, T_S)}{T_S} \tag{16}$$

where T_M is the length of $p_0(t)$, defined as the period of time in which a given percentage of the pulse energy is contained. Equation (16) indicates that the time of possible collision is equal to the correlator window ($2T_M+\varepsilon$), except when ($2T_M+\varepsilon$) is either 4 times greater than T_M or greater than T_S.

The next step for estimating BER is to define the shape of the probability function $P_{Zmui}(y,i)$. As shown in [11], the cumulative density function of MUI caused by one single interferer can be reasonably fitted by a linear function. Based on [11], we propose for $P_{Zmui}(y,i)$ a linear model including multiple interferers with different received powers (see Fig.1). $P_{Zmui}(y,i)$ is expressed by:

$$P_{Z_{mui}}(y,i) = \begin{cases} 1 & if\ y \le -Z_{\max}(i) \\ 1 - \frac{P_{CP}(i)}{2}\left(1 + \frac{y}{Z_{\max}(i)}\right) & if\ -Z_{\max}(i) < y \le 0 \\ \frac{P_{CP}(i)}{2}\left(1 - \frac{y}{Z_{\max}(i)}\right) & if\ 0 < y \le Z_{\max}(i) \\ 0 & if\ Z_{\max}(i) < y \end{cases} \tag{17}$$

where $Z_{max}(i)$ is the maximum value for Z_{mui} when i colliding pulses are present. For i collisions, the linear model in Fig. 1 indicates that an error occurs with probability 1 if the sum of Z_u and Z_n, i.e. y, is negative and lower than $-Z_{\max}(i)$. Such a probability of error decreases with y with a linear slope depending in turn on $P_{CP}(i)$. Obviously, the probability of bit error is 0 when y is positive and higher than $Z_{max}(i)$. In this case, in fact, MUI does not provoke an error since Z_{mui} is below the y term. Power control is not assumed at RX, and therefore $Z_{max}(i)$ varies. We suppose that:

$$Z_{max}(i) = \sum_{j=1}^{N_i} \left(\left\lceil \frac{i - j + 1}{N_i} \right\rceil \sqrt{E_S^{(j)}} \right) \tag{18}$$

where $E_S^{(1)}, E_S^{(2)}, ..., E_S^{(Ni)}$ are the interfering energies $E^{(1)}, E^{(2)}, ..., E^{(Ni)}$ of (4), sorted so that $E_S^{(j)} \ge E_S^{(j+1)}$ for $j \in [1, N_i - 1]$. Eq. (18) is an upper BER bound since $Z_{max}(i)$ is estimated by privileging users with highest interfering energies. When substituting (17) into (14), and after a few simplifications, one can find the following BER upper bound:

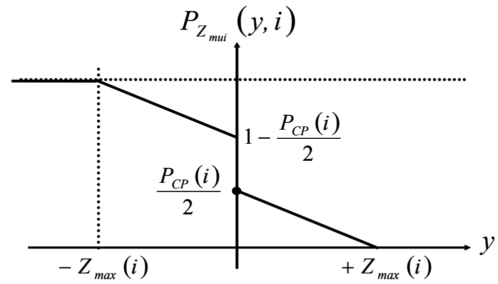

Figure 1. Linear model for function $P_{Zmui}(y,i)$.

$$\mathrm{BER}_{\mathrm{UPBOUND}} = \frac{1}{2} erfc\left(\sqrt{\frac{1}{2} \frac{N_S E_u}{\mathcal{N}_0} (1 - R_0(\varepsilon))} \right) + \sum_{i=0}^{N_i N_S} \frac{P_{CP}(i)}{2} \Omega\left(\frac{N_S E_u}{\mathcal{N}_0}(1 - R_0(\varepsilon)), \frac{Z_{\max}(i)^2}{N_S \mathcal{N}_0 (1 - R_0(\varepsilon))} \right) \tag{19}$$

where

$$\Omega(A,B) = \frac{1}{2} erfc\left(\sqrt{\frac{A}{2}} - \sqrt{\frac{B}{2}} \right) + \frac{1}{2} erfc\left(\sqrt{\frac{A}{2}} + \sqrt{\frac{B}{2}} \right) - erfc\left(\sqrt{\frac{A}{2}} \right)$$

$\mathrm{BER}_{\mathrm{UPBOUND}}$ as in (19) includes a first term of thermal noise that only depends on signal to thermal noise ratio at RX input, and a second term accounting for MUI. Note that for computing (19), no additional information with respect to the BER computation under the SGA is requested.

IV. Simulation Results

Simulation of a network of 4 nodes provided the results presented in Figs.2 and 3 for two reference signal formats. In both cases, 4 users are considered (N_i=3), and power control is assumed at RX. In the case of Fig. 2, transmitted signals have $N_S = 2$ and $T_S = 25$ns, leading to R_b=20 Mb/s (signal format *A*). In Fig. 3, transmitted signals $N_S = 4$ and $T_S = 25$ns, leading to R_b=10 Mb/s (signal format *B*). In both cases, $p_0(t)$ is the second derivative Gaussian waveform [13], with $T_M = 1$ ns and $\varepsilon = 1$ ns. Performance is expressed by BER vs. signal to noise ratio $E_b/\mathcal{N}_0$, where E_b=$N_S E_u$ is the received energy per bit. Note that Figs. 2 and 3 are computed for same E_b, meaning that E_u is different in the two figures. Thus, one should not be surprised if performance seems to degrade from $N_S = 2$ to $N_S = 4$ since E_u is smaller for $N_S = 4$. BER estimates based on Pulse Collision (squares) are plotted against simulation values (solid line) and SGA values (circles). Note that Pulse Collision values very well fit simulation data, while SGA underestimates BER.

Figure 4 compares Pulse Collision vs. SGA for to signal format *A* and *B*, when increasing N_i. Observe that SGA tends to Pulse Collision for high N_i, that is, when the number of collisions at the receiver input justifies the application of the central limit theorem. Preliminary investigations obtained by removing the hypothesis of power control or by varying N_i and N_S seam to lead to similar network behavior.

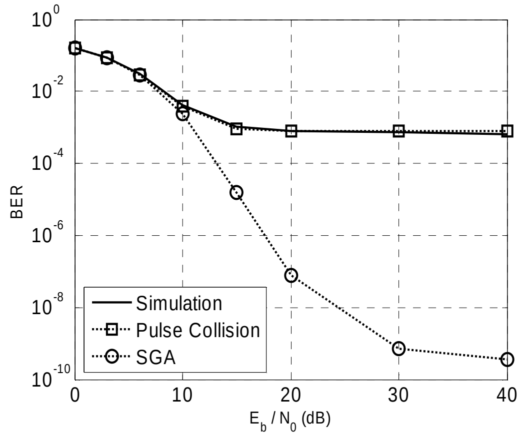

Figure 2. BER vs. $E_b/\mathcal{N}_0$ with signal format *A* ($N_S = 2$, $T_S = 25$ ns) and N_i=3.

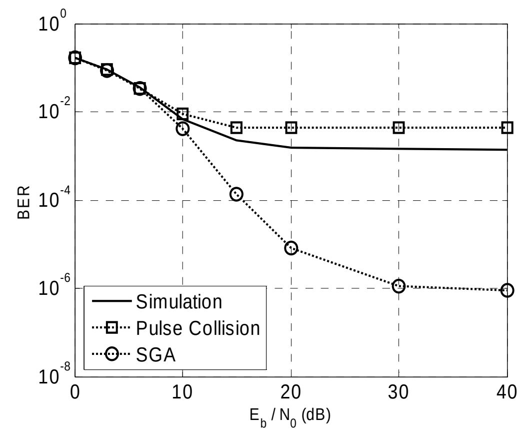

Figure 3. BER vs. $E_b/\mathcal{N}_0$ with signal format *B* ($N_S = 4$, $T_S = 25$ ns) and N_i=3.

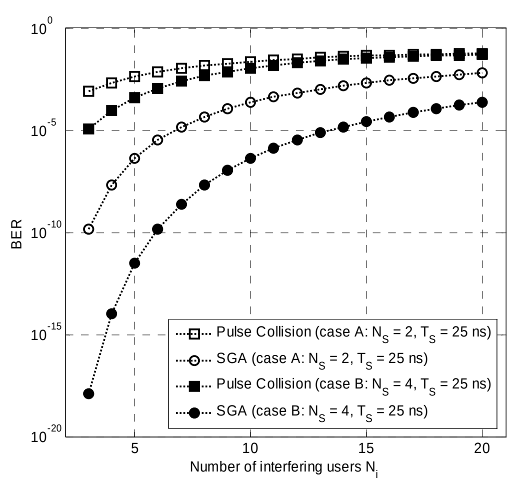

Figure 4. Theoretical BER vs. number interfering users N_i for the two models (Pulse Collision and SGA) and for the two different signal formats (signal format *A* and *B*).

V. CONCLUSIONS

The present work extends [12] by redefining the event of pulse error based on a complete receiver structure definition. The proposed receiver model includes soft detection. A refined definition of the bit error probability when one or more collisions have occurred, which also incorporates the effect of thermal noise in the receiver, is introduced. Results show that the proposed model provides an excellent fit of simulated performance measurements, and proves to be particularly effective in those cases where SGA fails to predict network behavior. This model does not require to specify any additional system parameters compared to SGA.

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