A Model for Cellular Systems with Broadband Users: Effect of Users Speed on Resource Allocation Efficiency

Gabriele Lupo(1), **Nicola Blefari-Melazzi**(2), **Roberto Cautelier**(1), **Maria-Gabriella Di Benedetto**(1)

(1) Università degli studi di Roma La Sapienza, Dipartimento INFOCOM,
Via Eudossiana 18, 00184 Roma, Italy.
e-mail: gaby@acts.ing.uniroma1.it
(2) Università di Perugia, DIEI, Via G. Duranti 93, 06125 Perugia, Italy.

Abstract

An analytical model of a cellular system was developed with the aim of understanding the effect of the degree of user mobility on resource allocation efficiency, in terms of packet dropping. We refer to a multi-cell environment, where a fixed portion of the channel is assigned to each cell. The dynamic behaviour of the cell occupancy states is modelled for the case of a low number of broadband users making inter-cell handover. The model gives a useful insight about the decrease of the packet dropping probability as the degree of mobility increases.

1. Introduction

The present work was motivated by the analysis of simulation results of a particular system with a low number of broadband moving users (6–10 per cell) presented in [1], which indicated a dependence of packet dropping probability on the degree of user mobility. Indeed, as the probability of making HandOver (HO) increases, the packet dropping diminishes. The system simulated in the above mentioned paper was based on an extension of the MEDIAN–ACTS project architecture to a multi–cell case. This work is referred to the same scenario.

The MEDIAN system is designed for multimedia communications in a Wireless Local Area Network (WLAN), in which broadband users transmit ATM–like packets using a TDD–TDMA–OFDM technique [2]. High traffic is offered to each Base Station (BS) by a small number of users.

This work proposes a theoretical model to study the effect of the degree of mobility on system performance, in terms of packet dropping probability. The uplink channel is considered and the analysis is carried out by considering two cells, each one controlled by a BS. Each Mobile Station (MS) is provided with a finite length k buffer indicated as *user buffer*. The transition of each user from one cell to the other is assumed to be instantaneous.

The theoretical model is represented by a particular Markov–chain, which takes into account the variations of the number of users served by each BS and the user buffer occupancy states, as packets are generated and transmitted over the multi–user channel.

In Section 2 the meaning of the states constituting the chain and the hypotheses determining the Markovianity of the model will be described in a detailed manner. In Section 3 and 4, we will focus on the special case of two users and some related analytical results will be presented. Finally, Section 5 will be dedicated to the conclusions.

2. The Markov–chain model of the system with broadband moving users

2.1 Model description

The proposed model consists of a continuous–time Markov chain, subdivided into a set of *sub–chains*. Each sub–chain corresponds to a specific distribution of users among the two cells. Indicating by **N** the total number of users, the first sub–chain represents the case of all users in one cell, labelled (N;0); the second, that of one user in a cell and the remaining (N–1) in the other, labelled (N–1;1), and so on, up to the last sub–chain (N/2;N/2), representing that of uniformly distributed users. Thus, the number of sub–chains is (N/2+1).

In each sub–chain, each state corresponds to a different buffer occupancy condition, i.e. two states are different if the number of packets contained in each of the N buffers differ in at least one buffer. Consequently, the number of states in each sub–chain is $(k+1)^N$.

A state is characterised by an ordered N–uple of integers, corresponding to the N queues lengths. As for the notation, an N–uple such as (A_i B_j C_l D_m) represents the state of a sub–chain of a system with four users (indicated by A,B,C,D) characterised by their buffer occupancy (indicated by the sub–index i,j,l,m). As can be observed, all sub–chains are composed of "similar" states, since two different sub–chains have always two states representing the same buffer occupancies, although corresponding to different distributions of the users among the two cells.

2.2 Transitions between states

Transitions between states occur either when a packet is generated, or when a packet is transmitted, or when a user makes HO. In the first two cases, the transition occurs inside a sub–chain, while in the third case the transition is between "similar" states of different sub–chains. An example of the possible transitions between states belonging to the sub–chains (N/2+1;N/2–1) and (N/2;N/2) are illustrated in Fig.1.

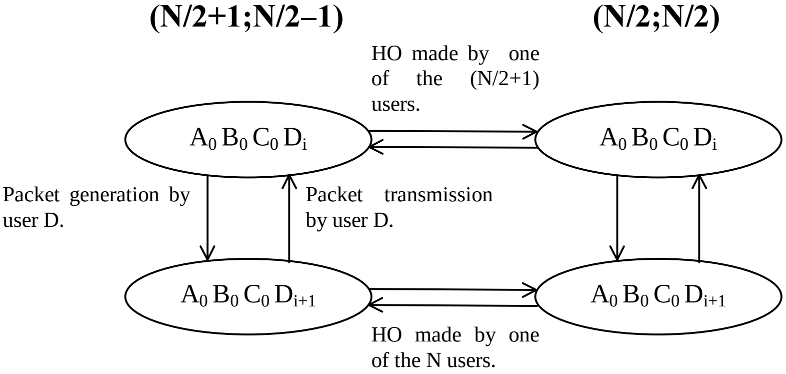

Figure 1: Possible transitions between a sub-set of states of the sub-chains respectively associated to the case of (N/2+1) users in a cell (N/2-1 in the other cell) and N/2 users in both cells

The number of sub–chains constituting the model is (N/2+1); each one represents the distribution of users between the two cells independently from which cell serves more or less users, because the overall behaviour of the system is the same in both cases. Since each sub–chain is composed of $(k+1)^N$ states, the total number of states, Σ, is given by:

$$\Sigma = (k+1)^{\mathrm{N}} \cdot \left(\frac{\mathrm{N}}{2} + 1 \right) \tag{1}$$

The total number of states is finite, consequently the chain is ergodic and completely described by the steady–state probabilities p_i ($i = 1,\ldots, \Sigma$), which can be computed from the $\Sigma\times\Sigma$ transition rates matrix [Q], by solving the following system of Σ linear equations:

$$[\mathrm{Q}]^{\mathrm{T}}\underline{\mathrm{p}}=\underline{0} \tag{2}$$

coupled with the probability conservation equation:

$$\sum_{i} \mathrm{p}_{i} = 1 \tag{3}$$

2.3 Hypotheses

In order to guarantee the Markovianity of the proposed model, the following hypotheses must be made:

- The packet generation be a Poisson process, so that the interarrival times be exponentially distributed random variables with average rate λ.
- The service times be exponentially distributed random variables with average rate μ_i in the states of the i–th sub–chain. All states belonging to the same sub–chain have the same average service rate. This hypothesis corresponds to assuming that the service time is independent from the number of packets waiting to be transmitted, and that it only depends on the distribution of users among the BSs. Service time is defined as the time between two subsequent packet transmissions, for a given distribution.
- The time between subsequent HO effectuated by a user is an exponentially distributed random variable with average rate $\lambda_{\mathbf{m}}$, equal to the inverse of its average dwell–time in a cell. The parameter λ_m corresponds to the degree of user mobility, and is directly related to the user speed.

The validity of the proposed model extends beyond the particular multiple access scheme adopted. The access is only supposed to be collision–free, centrally controlled and characterised by allocations rapidly varying depending on the offered traffic.

3. Model for N=2

Since the number of states constituting the chain tends to be extremely high as N and k are increased, this section is devoted to a performance analysis for the simple case of N=2 and k such that steady–state probabilities can be easily calculated. The highest value of k in this work is 16, for which the number of states is 578 and the dimension of the matrix [Q] is 578×578. This case offers useful insights while still avoiding onerous computations. On the other hand, the analysis could be extended to greater values of N and k through the use of appropriate numerical analysis procedures.

Figure 2 illustrates the model for N=2 (users labelled A,B) and k=2. The extension to higher values of k is straightforward.

Service rate is denoted by μ_2 for the sub–chain (2;0) and μ_1 for the sub–chain (1;1).

In order for the packet dropping probability not to be excessively high and to highlight the effect of saturation when the users are concentrated in the same cell, the following relation will be considered:

$$\mu_2<\lambda<\mu_1 \tag{4}$$

When the users are served by the same BS, service time is doubled with respect to the case of uniform distribution and, consequently, we have the following relation between μ_2 and μ_1:

$$\mu_2 = \frac{\mu_1}{2} \tag{5}$$

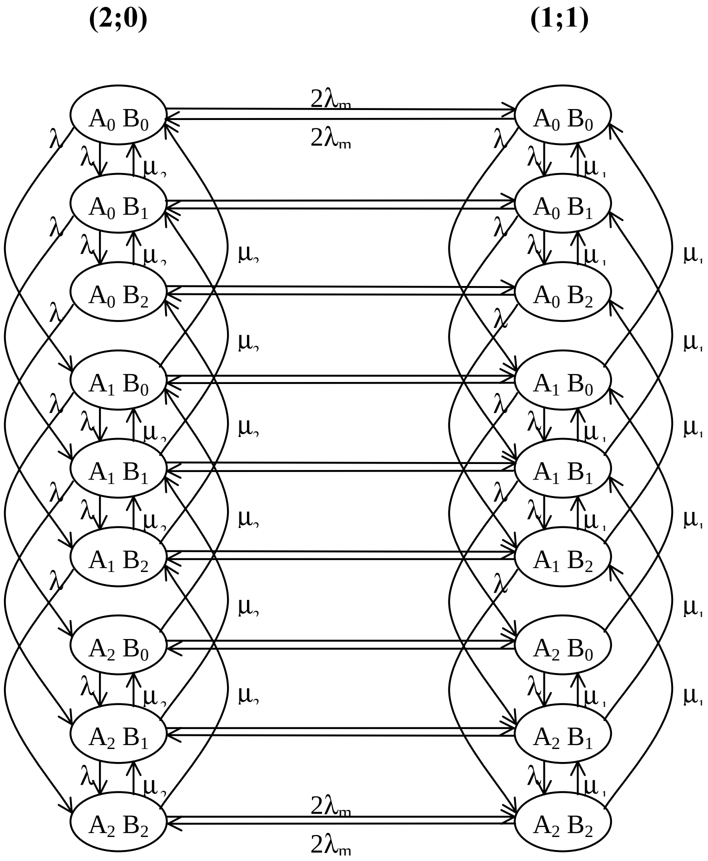

Figure 2: State transition diagram for N=2 and k=2. The two sub-chains correspond to the case of: two users in one cell, (2;0), and one user in each cell, (1;1)

4. Results

The effect of the degree of mobility (λ_m) on packet dropping probability for different values of the average service rates (μ_1 and μ_2), fulfilling relation (5), is shown in Fig.3.

The packet dropping probability is the same for all users; it can be calculated as the sum of the state probabilities of those states in the chain corresponding to a saturated buffer with regards to a single user. In fact, all users have identical statistical characteristics, and the packet generation processes of different users are independent from each other and from the chain states.

All the parameters are normalised to the frame duration. The following value is adopted for the average packet generation rate, according to the 3 *Mbps* video user model described in [3] and to the frame structure used in [1]: $\lambda = 3$ *packets/frame*.

Figure 3 highlights the reduction of packet dropping as the degree of mobility increases, which corresponds to an increase of λ_m. This effect can be explained as follows: as the speed increases, the probability of worst condition, i.e. when the users are in a single cell and the user buffer tends to saturation, decreases. Note that for $N=2$, there are only two possible distributions: the best case of uniform distribution (one user in each cell) and the worst case (two users in one cell).

The values have been selected only for descriptive purposes, since we are interested in the qualitative system behaviour. As expected, packet dropping probability values are lower for higher values of the average service rates, however they tend to be similar for higher user speed.

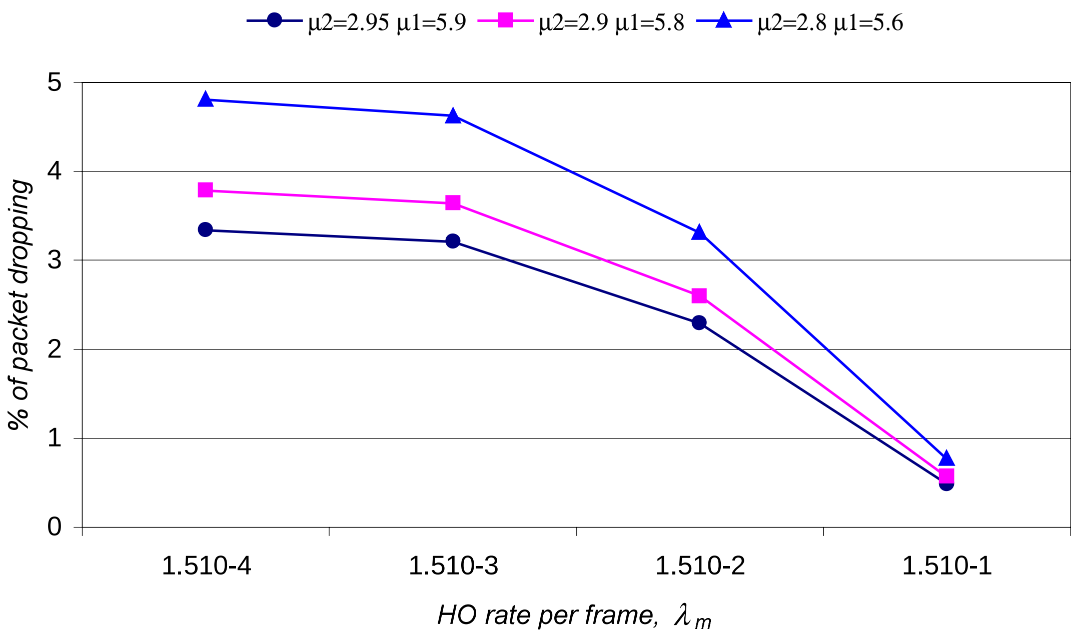

Figure 3: Packet dropping percentage vs. degree of user mobility. Parameters are expressed in *packets/frame*

5. Conclusions

In this paper, an analytical model of a cellular system, based on Markov chain theory, is proposed. The model helps understanding the effect of the degree of user mobility on packet dropping and gives a useful insight about the decrease of the packet dropping probability as the degree of mobility increases. Results for the particular case of two broadband users making inter–cell handover have been shown, although the model is valid even for a higher number of users.

6. References

[1] F. Malena, M. Del Buono, M.G. Di Benedetto, "Dynamic resource allocation with a novel handover procedure for application in a broadband system", in *Proc. of 3rd ACTS Mobile Communication Summit,* June 1998, Rhodes, Greece.

[2] European Community ACTS MEDIAN project AC006, "Wireless Broadband CPN/LAN for Professional and Residential Multimedia Applications", Final System Design, Deliverable MD3, CEC reference #AC006/MED/PAR/DS/R/054/b1, Dec. 1996.

[3] M. Del Buono, L. Hanzo, P. Cherriman, "Oscillation–scaled histogram–based Markov modelling of video codecs", in *Proc. of PIMRC'97*, Sept. 1997, Helsinki, Finland.