

Multi-hop Cognitive Radio Networking through Beamformed Underlay Secondary Access

Auon Muhammad Akhtar¹, Luca De Nardis², Mohammad Reza Nakhai¹, Oliver Holland¹,
Maria Gabriella Di Benedetto² and A. Hamid Aghvami¹

¹Institute of Telecommunications, King's College London, Strand, UK, WC2R 2LS

²DIET Department, Sapienza University of Rome, Rome, Italy

{auon.akhtar, reza.nakhai, oliver.holland, hamid.aghvami}@kcl.ac.uk, {luca.n, dibenedetto}@newyork.ing.uniroma1.it

Abstract—This paper introduces a transmit beamforming strategy taking into account the positions of primary, secondary victim and intended secondary receivers, to achieve underlay secondary access in multihop cognitive radio networking. The transmit beamforming strategy defines a novel path optimization scheme that deviates from a preselected path given by the routing module, based on local information and according to a relay selection metric. This metric is designed to improve both coexistence with primary/secondary victim receivers and performance of the secondary cognitive network. Simulations compare the proposed strategy with a baseline solution that does not adopt beamforming, and with a strategy that applies beamforming on each hop without modifying the original path. Results show that the proposed strategy is capable of improving coexistence with primary/secondary victims, and highlight that a trade-off exists between the meeting of coexistence constraints and maximisation of secondary network performance.

Index Terms—Beamforming, cognitive radio, multi-hop, power control, routing.

I. INTRODUCTION

So far, most of the research on cognitive radios has been focused on single-hop scenarios, tackling physical (PHY) layer and/or medium access control (MAC) layer issues [1], [2]. However, recent research findings have highlighted the potentials of multi-hop cognitive radio networks [3]. The cognitive paradigm can be applied to different scenarios of multi-hop wireless networks, one such scenario being the cognitive radio ad hoc network which consists of CR nodes which communicate with each other in a peer to peer fashion through ad hoc connections [4]. To fully realize the potential of such networks, cross-layer design issues must be addressed, for example, the routing decisions at the network layer should be made in conjunction with the PHY layer characteristics.

In the above framework, the present work focuses on the specific case of an underlay secondary network, adopting transmit beamforming in order to guarantee coexistence with the primary nodes. Although beamforming has been proposed in the past as a way to improve the capability of secondary network nodes to meet QoS requirements set by a primary network [5], this paper goes beyond previous attempts by taking into account beamforming in the selection of relays in multi-hop connections. The proposed approach defines a metric that, relying on position information available at local level, modifies the path originally selected by the routing algorithm so to improve network performance while guaranteeing

coexistence requirements; potential solutions for retrieving the required position information that take advantage of the same hardware used to implement the beamforming scheme are also discussed. The proposed strategy is compared with previous work that adopted beamforming in the secondary network without changing the path.

The paper is organized as follows. Section II presents the system model and network architecture; Section III presents the transmit beamforming strategy while the path optimization strategy is described in Section IV. Simulation results are presented in Section V while Section VI draws conclusions.

II. THE NETWORK ARCHITECTURE & SYSTEM MODEL

We consider a multi-hop cognitive radio network where the primary and the secondary nodes are randomly distributed within a specified region. Our goal is to route data between the secondary source and destination nodes in an energy efficient manner while minimising the co-channel interference amongst the secondary nodes. Furthermore, we aim to keep the interference imposed on the primary users by the secondary transmissions within the allowed interference shaping margins (ISM) required by each primary user. It is assumed that each secondary node is equipped with a uniform linear array (ULA) with N transmit antennas with an element spacing of half a wavelength. For reception, both the primary and the secondary nodes utilize a single receive antenna. Thus, the considered scenario forms a multiple-input single-output (MISO) communication link. Such scenario makes sense in a multi-hop network, as the adoption of receive beamforming would make the implementation of broadcast and flooding procedures overly complicated.

Denoting the beamforming weight vectors by $\mathbf{w}_{ir} \in \mathcal{C}^{N \times 1}$, where x_i represents the secondary transmitter while y_r represents the secondary receiver, the transmitted signal is given as $\Psi_i = \mathbf{w}_{ir}s_r$, where s_r is the data symbol intended for the receiver y_r . The signal received at y_r can be expressed as $\Upsilon_r = \mathbf{h}_{ir}\mathbf{w}_{ir} + n_r$, where n_r is the receiver noise with power P_n while $\mathbf{h}_{ir} \in \mathcal{C}^{1 \times 1N}$ is the spatial channel response vector between transmitter x_i and receiver y_r . \mathbf{h}_{ir} can be written as $\mathbf{h}_{ir} = [h_{i,r,1} \ h_{i,r,2} \ \dots \ h_{i,r,N}]$, where $h_{i,r,k}$, $k \in [1 \dots N]$, models the channel between the k^{th} element of the transmitter x_i and the receiver y_r and can be written as $h_{i,r,k} = e^{j\frac{2\pi\Delta}{\zeta}(k-1)\sin(\theta_{ir}+\phi)} A\sqrt{1/L_{ir}}$, where Δ is the

antenna element spacing, ζ is the wavelength while θ_{ir} is the angle-of-departure (AoD) relative to the array antenna broadside; ϕ represents a small deviation from θ_{ir} with normal distribution, i.e., $\phi \sim \mathcal{N}(0, \sigma^2)$, where σ represents the angular spread of local scatters surrounding node y_r . Finally, A represents the fading coefficient between the transmitter and the receiver while L_{ir} is the distance dependant pathloss.

III. TRANSMIT BEAMFORMING STRATEGY

We aim to minimize the total transmitted power subject to certain constraints. These constraints are the interference margins of the primary nodes and minimum signal-to-noise-ratio (SNR) requirements of the secondary nodes. Instead of using instantaneous channel state information (CSI), we use the second order statistics of the channel state information at the transmitting nodes, i.e., $\mathbf{R}_{ir} = \mathbf{E}[\mathbf{h}_{ir}^H \mathbf{h}_{ir}]$ and $\mathbf{R}_{pm} = \mathbf{E}[\mathbf{h}_{pm}^H \mathbf{h}_{pm}]$, $1 \leq m \leq M$, where \mathbf{R}_{ir} and \mathbf{R}_{pm} are the channel autocorrelation matrices for the secondary receiver r and m^{th} primary node, respectively. It is assumed that the secondary transmitters have knowledge about the locations of all the primary nodes within their transmission range. Instead of using a large number of samples to obtain the second-order statistics, we use the expression derived in [6] to obtain these statistics directly. Thus, the $(k, l)^{\text{th}}$ entry in the matrix \mathbf{R} can be written as $\mathbf{R}(k, l) = \frac{A^2}{L} e^{j \frac{2\pi\Delta}{\zeta} (l-k) \sin(\theta)} e^{-2[\frac{\pi\Delta\sigma}{\zeta} (l-k) \cos(\theta)]^2}$. The transmit power at secondary node i is minimized as

$$\begin{aligned} & \text{minimize} && p_i^t \\ & \text{subject to} && SNR_r \geq \gamma_r, \\ & && I_m \leq \varphi_m, 1 \leq m \leq M, \end{aligned} \quad (1)$$

where p_i^t is the power transmitted from node i , SNR_r is the received SNR at node r , γ_r is the minimum required SNR for node r , I_m is the interference caused to primary node m while φ_m is the upper bound on maximum allowed interference towards the primary node m due to secondary transmissions. SNR_r is given as $SNR_r = \frac{p_r}{P_n}$, where p_r is the power received at node r and is given by $p_r = \mathbf{w}_{ir}^H \mathbf{R}_{ir} \mathbf{w}_{ir}$. Similarly, the interference exerted upon the m^{th} primary user can be written as $I_m = \mathbf{w}_{ir}^H \mathbf{R}_{pm} \mathbf{w}_{ir}$. Finally, the total transmission power p_i^t can be written as $p_i^t = \mathbf{w}_{ir}^H \mathbf{w}_{ir}$. Thus, the problem in (1) can be rewritten as

$$\begin{aligned} & \text{minimize} && \mathbf{w}_{ir}^H \mathbf{w}_{ir} \\ & \text{subject to} && \mathbf{w}_{ir}^H \mathbf{R}_{ir} \mathbf{w}_{ir} \geq \gamma_r P_n, \\ & && \mathbf{w}_{ir}^H \mathbf{R}_{pm} \mathbf{w}_{ir} \leq \varphi_m, 1 \leq m \leq M \end{aligned} \quad (2)$$

Since the problem in (2) is non-convex, it has to be converted into convex SDP form, which can be solved by an SDP solver like SeDuMi. To do this, we define a new matrix \mathbf{F}_{ir} as $\mathbf{F}_{ir} = \mathbf{w}_{ir} \mathbf{w}_{ir}^H$. With this matrix, (2) can be written as

$$\begin{aligned} & \text{minimise} && \text{tr}[\mathbf{F}_{ir}] \\ & \text{subject to} && \text{tr}[\mathbf{R}_{ir} \mathbf{F}_{ir}] \geq \gamma_r P_n \\ & && \text{tr}[\mathbf{R}_{pm} \mathbf{F}_{ir}] \leq \varphi_m, 1 \leq m \leq M, \\ & && \mathbf{F}_{ir} = \mathbf{F}_{ir}^H \geq 0. \end{aligned} \quad (3)$$

Notice that we have used the rotation property of the trace operator, i.e., $\text{tr}[\mathbf{AB}] = \text{tr}[\mathbf{BA}]$ to arrive at $\mathbf{w}_{ir}^H \mathbf{R}_{ir} \mathbf{w}_{ir} = \text{tr}[\mathbf{R}_{ir} \mathbf{w}_{ir} \mathbf{w}_{ir}^H] = \text{tr}[\mathbf{R}_{ir} \mathbf{F}_{ir}]$. Further details about the above derived formulation can be found in [1]. As a final note on the above beamforming strategy, in order for it to be applied, an estimation of the direction towards the intended receiver and victim primary and secondary receivers must be available. The impinging directions of the secondary users can be estimated by the secondary BS using the algorithm described in [7].

IV. MULTI-HOP COGNITIVE RADIO ROUTING SCHEMES WITH HOP-BY-HOP BEAMFORMING

We now utilize transmit beamforming to design a routing algorithm for multi-hop cognitive radio network. The objective of the algorithm is three-fold: 1) To minimize the end to end power consumption; 2) To minimize the co-channel interference generated within the secondary network; 3) To minimize the number of primary interference constraint violations.

To achieve the goals set above, we adopt a centralized approach whereby the optimal power saving route is initially calculated through Dijkstra's algorithm by using the point to point link costs without beamforming, which can be written as $LC(i, r) = \frac{\gamma_r P_n}{|h_{ir}|^2}$, where $LC(i, r)$ represents the link cost between transmitter x_i and receiver y_r while all the other parameters have already been defined in Section II.

After this initial step, the algorithm modifies the selected route by using a new cost metric which we introduce later in this section. To ensure that the modified route does not deviate too much from the optimal power saving route, the cost metric is used only on alternate hops, for example, for every odd numbered hop of the optimal route, the hop destination is selected based on the proposed cost metric, while the destinations of the even numbered hops remain unchanged. We now propose a cost metric which is used to select the node which is most suitable to act as a relay. The proposed metric takes into account the potential impact of the selection of a relay on the primary receivers and other secondary nodes, within the transmission range of the source and the candidate relay node. In the following, we refer to the source, relay and destination nodes as S , R and D , respectively. A terminal R will only be eligible as a relay if it meets all the following conditions:

- 1) S does not violate the interference constraint of any of the primary users when it transmits data to R using beamforming;
- 2) R does not violate the interference constraint of any of the primary users when it transmits data to D using beamforming;
- 3) The position of R is such that the distance between R and D , indicated as $dist_{RD}$, is not larger than the distance from S to D , $dist_{SD}$. This condition ensures physical connectivity between the selected relay and D and it ensures that the algorithm remains loop free.

The above description translates into the cost $Cost(S, D, R)$ associated to the generic terminal R as a potential relay between S and D defined by eq.(4) where:

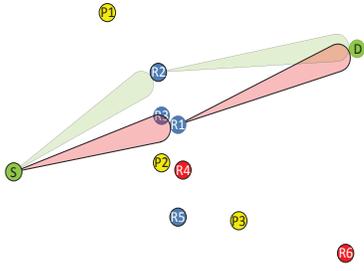


Fig. 1. Demonstration of the proposed cost metric.

- 1) N_k^S is the number of secondary terminals within the transmission range of terminal k ;
- 2) N_k^P is the number of primary receivers within the transmission range of terminal k ;
- 3) $I(x, y, z)$ is the interference generated by x towards the generic receiver y (either primary or secondary) when transmitting to z .

The candidate relay node which gives the minimum value for the above cost function is selected as the next hop destination. Fig. 1 shows an example scenario where the green nodes are the secondary source and destination nodes, the yellow nodes are the primary nodes while the nodes labelled as R1,...,R6 are the candidate relay nodes. Amongst the candidate relay nodes, the nodes in blue are the eligible candidates while the nodes in red, i.e., R4 and R6 are not eligible to act as relays. R4 is rejected because S cannot transmit to R4 without violating the interference constraint of P2 while the distance between S and R6 is larger than the distance between S and D. The Dijkstra's algorithm selects R1 as the best option to act as a relay. However, it can be seen that if the data is transmitted to R1, a lot of interference is exerted upon R3. On the other hand, if R3 is selected as relay, interference will be exerted upon R1 when R3 forwards the data to D. Using the proposed cost metric, the best option here is to select R2 as relay.

V. PERFORMANCE EVALUATION

The performance of the proposed solution was evaluated by means of computer simulations executed by combining MATLAB and OMNeT++ as follows:

- 1) MATLAB was used to implement the transmit beamforming strategy introduced in Section III and to analyze the performance of the route optimization approach defined in Section IV by measuring the interference generated towards each secondary node as well as the average number of constraints set by primary receivers that are met.
- 2) OMNeT++ was used to test the proposed strategy in presence of actual packet transmissions in order to measure the impact of the proposed solution on throughput, moving from results generated in MATLAB.

A. Simulation scenario and setup

The MATLAB code was used to simulate a network of secondary nodes equipped with a ULA with $N = 8$ an-

tenna elements and a spacing between adjacent elements $d = 0.0625 m$, corresponding to half a wavelength for a carrier frequency $f_c = 2.4 GHz$, and capable thus to perform DOA estimation and beamforming. An angular spread $\Delta\theta = 2^\circ$ was introduced around the exact angle for each measurement. A noise power $P_n = -101 dBm$ was assumed at each receiver, while the pathloss exponent for propagation was set equal to $\alpha = 2$. MATLAB was used to solve the optimization problem of (3) by taking advantage of the SeDuMi solver provided by the cvx package [8], imposing an upper bound φ_m on the allowed interference towards the primary nodes and a minimum SNR level of $\gamma_r = 10 dB$ for all the secondary nodes. The following steps were executed in MATLAB for each run:

- 1) Generation of a topology composed of N_S secondary nodes and N_P primary nodes randomly deployed in an area of $X_{max} = 50 m$ by $Y_{max} = 50 m$ square meters;
- 2) Generation of N_{conn} connection requests in the secondary network with random source and destination nodes, random duration uniformly distributed between $minDuration$ and $maxDuration$ and random delay from the previous connection request from same source node uniformly distributed between $minDelay$ and $maxDelay$; then, for each connection request: a) Selection of the best path according to the minimum power routing strategy defined in Section IV; b) Optimization of the path according to the proposed metric, defined again in Section IV; c) Measurement of interference generated towards secondary nodes not involved in the connection with and without optimization; d) Measurement of number of primaries for which the constraint on the maximum interference value is met with and without optimization.
- 3) Export to file of the data required by OMNeT++, consisting of: a) primary and secondary network topology; b) the list of the N_{conn} generated connection requests, including source, destination and duration; c) original and optimized paths for each connection; d) the reduction in the interference $I(x, y, z)$ perceived in y guaranteed by the introduction of beamforming in the link from x to z , for all $x-z$ pairs involved in any connection, for both original and optimized paths.

The inputs generated in MATLAB were used in a simulated secondary network built in OMNeT++, with each secondary node characterized by the architecture shown in Fig. 2. With reference to such architecture, it should be noted that:

- the mobility and clustering modules were not activated, as a static network with flat organization was assumed.
- the positioning module was configured so to provide perfect position information about all network nodes.
- the application module for a generic node x was in charge of reading from file connection requests having x as source, and generate for each connection packets of size $appPacketSize$ bits spaced in time by a constant delay set to $applicationRate/appPacketSize$ (modeling thus

$$Cost(S, D, R) = \begin{cases} \sum_{i=1, i \neq R}^{N_S^S} I(S, R, i) + \sum_{k=1, k \neq D}^{N_R^S} I(R, D, k) & \text{if } \begin{cases} \sum_{l=1}^{N_S^P} \lfloor \frac{I(S, R, l)}{I_{MAX}(l)} \rfloor + \sum_{l=1}^{N_R^P} \lfloor \frac{I(R, D, m)}{I_{MAX}(m)} \rfloor = 0 \\ dist_{RD} \leq dist_{SD} \end{cases} \\ +\infty & \text{otherwise} \end{cases} \quad (4)$$

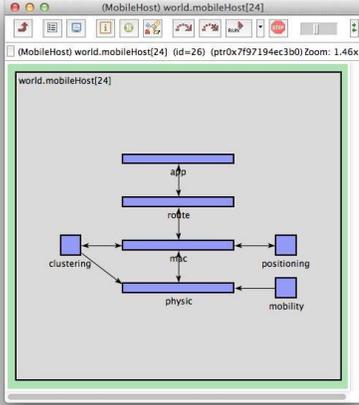


Fig. 2. Secondary node architecture implemented in OMNeT++.

- a Constant Bit Rate (CBR) packet stream) for a time equal to the connection duration read from file;
- the routing module for a source node x , upon receiving from the application module the first packet of a connection, was in charge of a) loading from file the corresponding end-to-end path determined in MATLAB, b) record such path in each packet; the routing module of intermediate nodes took care of forwarding the packet towards the destination by reading the next hop from the packet itself, while routing module of a destination node simply forwarded the packet to application module.
- the MAC module implemented a simple Aloha protocol without retransmission, taking care of immediately forwarding packets received from the routing module to the physical layer module and viceversa.
- the physical layer module had the responsibility of transmitting and receiving packets taking into account path loss, propagation delays and interference generated by packet collisions.

The impact of interference, in particular, was modeled with an accuracy significantly higher than what currently found in existing OMNeT++ frameworks, such as INET [9] and MixiM [10], in order to ensure a correct analysis of the impact of the proposed optimization on network performance. The simulator is in fact able to keep track of all transmitted packets and, for each packet reception, determines the interference level on a symbol by symbol basis (note that, as binary modulation was considered in all simulations, in the following bits will be

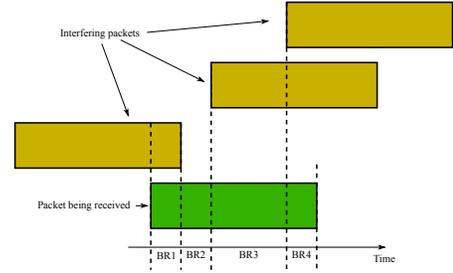


Fig. 3. Example of bit region identification during a packet reception in OMNeT++; 4 Bit Regions (BR1 to BR4) are identified based on the variations in the set of interfering packets.

considered in place of symbols). Consecutive bits subject to the same interference are grouped into so called bit regions: Fig. 3 shows an example of packet reception where four different regions are identified due to varying interference conditions. Next, for each bit region the average Bit Error Probability (BEP) is evaluated by adopting the Standard Gaussian Approximation for the interference power, and the number of bit errors is randomly determined according to the BEP. Finally the total number of bit errors generated is evaluated by summing up errors introduced in each bit region, and compared with the maximum number of errors admitted for the packet as determined by the adoption of a Reed-Solomon code with a coding rate $RS_{rate} = 0.835$ (corresponding to a correction capability roughly equal to 10% of the packet bits) in order to decide if the packet is correctly received or discarded. The following steps were executed in OMNeT++ for each run: 1) Loading of primary and secondary network topologies from file; 2) Loading of connection requests from file and for each request: a) Generation of packets; b) Forwarding of packets along the end-to-end path read from file; c) Measurement of end-to-end throughput and other relevant metrics; 3) Averaging of measured metrics. Table I presents the values for the simulation parameters.

B. Simulation Results

1) *Matlab results:* Fig. 4 shows the average interference imposed on the secondary nodes when the data is routed between the secondary source and destination nodes. To ensure continuity of the simulations, the constraint on primary interference is relaxed if the cost of (4) is $+\infty$ for all the secondary nodes within the transmission range of the transmitter for a specific hop. As can be seen from the figure, the optimized routing with beamforming, i.e., routing with the proposed cost metric, gives the best performance in

TABLE I
SIMULATION SETTINGS

Parameter	Value(s)
Number of secondary nodes N_S	50
Number of primary nodes N_P	from 10 to 50
Number of connection requests per run N_{conn}	1000
Minimum connection duration $minDuration$	25 s
Maximum connection duration $maxDuration$	75 s
Min delay between connection requests $minDelay$	50 s (High Traffic) / 500 s (Low Traffic)
Max delay between connection requests $maxDelay$	100 s (High Traffic) / 750 s (Low Traffic)
Transmission rate at physical layer	1 Mb/s
Maximum transmission power for secondary nodes	1 μW
Application packet length $appPacketSize$	512 bits
Application source rate $applicationRate$	320 kbit/s

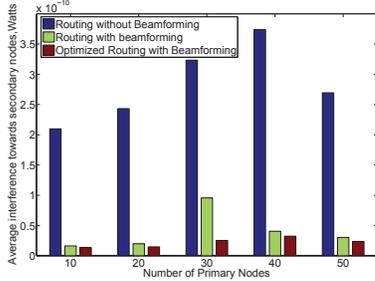


Fig. 4. Average total interference exerted upon the secondary nodes between source and destination.

terms of interference imposed within the secondary network. As expected, routing without beamforming gives the worst performance. Furthermore, it must be mentioned here that to compare the performance of routing with beamforming and optimized routing with beamforming, one must also consider the number of primary constraint violations, since we relax the primary interference constraint when none of the secondaries is able to satisfy this constraint. To make this comparison, Fig. 5 shows the number of primary constraint violations for different number of primary nodes.

From Fig. 4 and Fig. 5, it can be seen that the difference in performance between the optimized and non-optimized routing with beamforming in Fig. 4 is large when the corresponding difference in primary violations in Fig. 5 is relatively small, e.g., the performance when the number of primary nodes is 30. Otherwise, when the difference in performance in Fig. 4 is small, the difference in the number of primary constraint violations is relatively large. In order to have a fair comparison between the two, the number of primary constraint violations for routing with beamforming and optimized routing with beamforming should be forced to be the same.

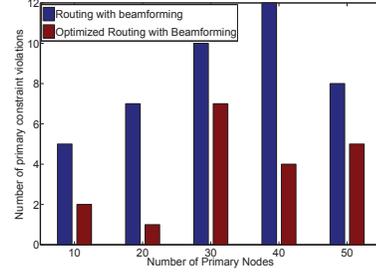
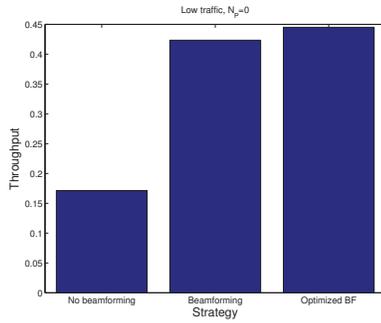


Fig. 5. Number of primary constraint violations

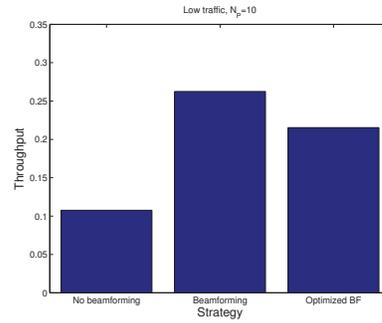
2) *OMNeT++ results*: OMNeT++ simulations considered the following four different scenarios, obtained by varying the traffic load in the secondary network and the number of primary nodes:

- *Low traffic, free network* - Low traffic (obtained by setting the $minDelay$ and $maxDelay$ variables to the corresponding values in Table I) and no primary nodes;
- *Low traffic, constrained network* - Low traffic and $N_P = 10$ primary nodes;
- *High traffic, free network* - High traffic (obtained by setting the $minDelay$ and $maxDelay$ variables to the corresponding values in Table I) and no primary nodes;
- *High traffic, constrained network* - High traffic and $N_P = 10$ primary nodes.

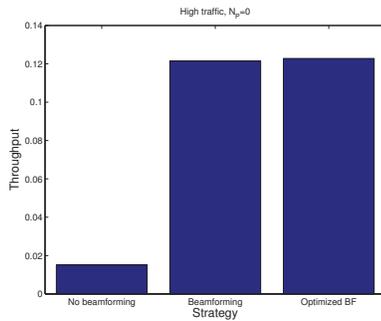
The throughput, defined as the ratio between end-to-end received packets and generated packets was measured in the four scenarios above for the three strategies previously introduced in the paper. Fig. 6(a) presents the throughput in the case of the *Low traffic, free network* scenario. The figure shows that in this scenario the optimization in the routing path leads to an increase in throughput, as on each other hop the strategy is able to select the node that provides the lowest amount of interference to neighboring nodes, thus increasing the probability of correct packet reception throughout the network. Moving to the *Low traffic, constrained network* scenario, Fig. 6(b) shows that the introduction of constraints determined by the presence of a significant number of primaries has the impact of reducing the gap between the two BF-based strategies, due to the fact that in several cases potential relays that would lead to lower interference in the secondary network are discarded as they do not satisfy the hard constraint on the level of interference towards one or more primary receivers. Fig. 6(c) shows how the throughput is affected in the *High traffic, free network*; results show how for all strategies performance is significantly reduced due to the higher number of collisions, and the corresponding higher average value of the interference power during packet reception. Finally, Fig. 6(d) shows results in the *High traffic, constrained network*, that introduces again the presence of the primary nodes; interestingly, results highlight that in this case the Optimized Routing with Beamforming leads to slightly worse results compared to simple Routing with Beamforming. However, as shown by Fig. 5, this comes together with a better coexistence capability with primary



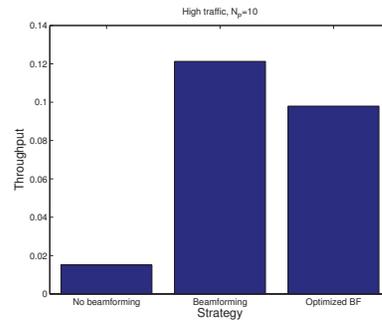
(a) Throughput in the *Low traffic, free network* scenario for the three considered routing strategies.



(b) Throughput in the *Low traffic, constrained network* scenario for the three considered routing strategies.



(c) Throughput in the *High traffic, free network* scenario for the three considered routing strategies.



(d) Throughput in the *High traffic, constrained network* scenario for the three considered routing strategies.

Fig. 6. Performance of the three routing strategies.

receivers, highlighting the presence of a trade-off between coexistence and secondary network performance.

VI. CONCLUSIONS

We focused on transmit beamforming and routing in a multi-hop, ad hoc cognitive radio network. After introducing the transmit beamforming strategy, we proposed a new cost metric which was used to design an optimized, beamforming based routing algorithm with three-fold objective: to minimize the end to end path power consumption; to minimize the co-channel interference imposed within the secondary network and to minimize the number of primary interference constraint violations. Simulation results from MATLAB confirmed that the optimized routing algorithm outperformed the original routing algorithm in terms of both, the interference generated within the secondary network and the number of primary interference constraint violations. The simulations carried out in OMNeT++ confirmed the improved throughput of the secondary network when no constraints from primary nodes are imposed, while they highlight a trade-off between coexistence capability and secondary network performance when the presence of primary nodes is taken into account.

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