Adaptive Multi-Coset Sampler

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Moreover, the current trends in wireless technology have increased the complexity of the receiver, more specifically its Analog to Digital Converter (ADC), due to the nature of broadband signals generated by certain applications, including communication in ultra wideband.

To sample a wideband signal with Nyquist rate will require a lot of effort and poses a major implementation challenge.
Sub-Nyquist Sampling

- [Mishali and Eldar, 2010] proposed for sparse multi-band signal, a sub-Nyquist sampler called Modulated Wideband Converter (MWC). MWC consists of several stages and each stage uses a different mixing function followed by a low pass filter and a low uniform sampling rate. This sampling technique shows that perfect reconstruction is possible when the band locations are known.
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- Multi-Coset (MC) sampling proposed in [Venkataramani and Bresler, 2001] is an effective way to reduce the frequency sampling for multi-band signals whose frequency support is a finite union of intervals.
Multi-Coset sampling

Over the recent years multi-coset sampling has gained fair popularity and several methods of implementing the MC sampling have been proposed.

- The most famous architecture is composed of several parallel branches, each with a time shift followed by a uniform sampler operating at a sampling rate lower than the Nyquist rate.

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x(t) \xrightarrow{p} \Delta_i \xrightarrow{t_n = nT_s} y_i(n)
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- [Domínguez-Jiménez and González-Prelcic, 2012] uses uniform samplers operating at different rates and is known as the Synchronous Multirate Sampling.
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- The most famous architecture is composed of several parallel branches, each with a time shift followed by a uniform sampler operating at a sampling rate lower than the Nyquist rate. The formula for the sampling time is:
  \[ t_n = nT_s \]
  \[ y_i(n) \]

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- The Dual-Sampling architecture is presented for multi-coset sampling by [Moon et al., 2012]. It is basically a subset of the Synchronous Multirate Sampling and uses only two uniform samplers.
MC sampling is a periodic non-uniform sampling technique which samples a signal at a rate lower than the Nyquist rate [Venkataramani and Bresler, 2001] [Rashidi Avendi, 2010].

**Explanation:**
- The analog signal $x(t)$ is sampled at Nyquist rate

**Example:**

![Graph showing multi-coset sampling](image)

- Analog signal
- Nyquist samples
Multi-Coset sampling in time domain

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- The analog signal $x(t)$ is sampled at Nyquist rate
- Divide the Nyquist grid into successive segments of $L$ samples each

**Example:** $L = 12$, 

![Graph showing multi-coset sampling](image)
Multi-Coset sampling in time domain

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**Explanation:**
- The analog signal $x(t)$ is sampled at Nyquist rate.
- Divide the Nyquist grid into successive segments of $L$ samples each.
- In each segment only $p$ samples out of $L$ are kept. Which $p$ samples? Described by the set $C$.

**Example:** $L = 12, p = 5, C = \{1, 5, 7, 9, 11\}$
The Fourier transform, $X_i(e^{j2\pi fT})$ of the sampled sequence $y_i[n]$ is related the Fourier transform, $X(f)$, of the unknown signal $x(t)$ by the following equation [Rashidi Avendi, 2010]:

$$y(f) = A_{cs}(f), f \in B_0 = \left[-\frac{1}{2LT}, \frac{1}{2LT}\right],$$  

(1)

$y(f)$ is a vector of size $p \times 1$ whose $i^{th}$ element is given by:

$$y_i(f) = X_i(e^{j2\pi fT}), f \in B_0, 1 \leq i \leq p$$

(2)
Le MC dans le domaine frequentiel (2)

\( \mathbf{A}_C \) est une matrice de taille \( p \times L \) dont l'élément \((i, l)^{th}\) est donné par :

\[
[\mathbf{A}_C]_{il} = \frac{1}{LT} \exp\left(\frac{j2\pi l c_i}{L}\right), \quad 1 \leq i \leq p, \quad 0 \leq l \leq L - 1
\]  

(3)

\( \mathbf{s}(f) \) représente le vecteur inconnu de taille \( L \times 1 \) avec le \( l^{th} \) élément donné par :

\[
\mathbf{s}_l(f) = \mathbf{X}(f + \frac{l}{LT}), \quad f \in \mathcal{B}_0, \quad 0 \leq l \leq L - 1
\]  

(4)

Les cellules actives \( \mathcal{K} = \{0, 1, 2, 3, 5\} \)

![Diagram of active cells](image)
Multi-Coset reconstruction

Matrix form, under-determined system

\[ y(f) = A_c S(f) \]

- **MC samples**
- Fourier transform
- **Sub-matrix of Fourier transform**
- **Nyquist samples**
- Fourier transform
Multi-Coset reconstruction

wholes detection

\[
y(f) = A_c S(f)
\]

MC samples
Fourier transform

sub-matrix of
Fourier transform

Nyquist samples
Fourier transform

\[
\begin{align*}
p \times 1 &= &p \times L \\
&= &L \times 1
\end{align*}
\]
Multi-Coset reconstruction

Resolvable system

\[ y(f) = A_c z(f) \]

MC samples
Fourier transform

sub-matrix of
Fourier transform

Nyquist samples
Fourier transform without wholes

\[ p \times 1 \]

\[ p \times |S| \]

\[ |S| \times 1 \]
MC Sampling parameters

MC sampling starts by first choosing:

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  $q = |\mathcal{K}|$ and $\mathcal{K} = \{k_r\}_{r=1}^{q}$, $k_r \in \mathbb{I} = \{0, 1, ..., L - 1\}$.
- The set $\mathcal{C} = \{c_i\}_{i=1}^{p}$ containing $p$ distinct integers form $\mathbb{I} = \{0, 1, ..., L - 1\}$. It should be noted that a good choice of the sampling pattern $\mathcal{C}$ reduces the margin of error due to spectral aliasing and sensitivity to noise in the reconstruction process.
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It is quite evident that once the sampling parameters (such as $p$) are selected, architecture of the MC sampler will remain unchanged irrespective of the input signal characteristics. Furthermore Optimal reconstruction that are proposed assume that the number of bands and the maximum bandwidth, a band can have, are known.
Adaptive Multi-Coset Sampler (AMuCoS)

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- They call it the *Adaptive Multi-Coset Sampler* or simply the AMuCoS sampler.
- It operates in blind mode, without any knowledge of the input signal’s spectral support and the number of bands.
Adaptive Multi-Coset Sampling

Introduction

Multi-Coset Sampling

Adaptive Multi-Coset Sampling

Conclusions
Non-Uniform Sampler Block (NUS)

We propose to design the NUS of our AMuCoS as a reconfigurable Additive Pseudo-Random Sampler (APRS) in conjunction with MC sampling. In APRS the $N$ sampling instants are defined as [Ben Romdhane, 2009]:

$$t_m = t_{m-1} + \tau_m = t_0 + \sum_{i=1}^{m} \tau_i, \quad 1 \leq m \leq N, \quad (5)$$

where $E[t_m] = mT$ and $var[t_m] = m\sigma^2$. For $N \geq 1$, $\{\alpha_m\}_{m=1}^{N}$ is a set of i.i.d random variables with density of probability $p_1(\tau)$, mean $T$ and variance $\sigma^2$. 
To design an APRS as a MC sampler for a given $C$ and $T$. We first defined the set of distances between two sampling instants by $T = \{\tau_i\}_{i=1}^{p}$ with $\tau_0 = c_1$, $\tau_i = c_{i+1} - c_i$ et $\tau_p = L + c_1 - c_{p-1}$. With $t_0 = T\tau_0$, equation (5) become:

$$t_m = T\sum_{i=0}^{m} \tau_i, \quad 0 \leq m \leq p, \quad \text{avec } \tau_i \in \mathbb{N}$$

The set of sampling instants $\{t_n\}_{n \in \mathbb{Z}}$ is non-uniform and periodic like the MC sampling.
Non-Uniform Spectral Sensing Block

Our System estimates the PSD of the non-uniformly sampled signal by using the Lomb-Scargle method [Lomb, 1976, Scargle, 1982]. Lomb-Scargle method evaluates the samples, only at times $t_n$ that are actually measured.

- Suppose that there are $N_s$ samples $x(t_n)$, $n = 1, ..., N_s$.
- The PSD estimate obtained from Lomb-Scargle Method is defined by:

$$P_{N_s}(\omega) = \frac{1}{2\sigma^2} \left\{ \frac{\left[ \sum_n (x(t_n) - \bar{x}) \cos \omega(t_n-\delta) \right]^2}{\sum_n \cos^2 \omega(t_n-\delta)} + \frac{\left[ \sum_n (x(t_n) - \bar{x}) \sin \omega(t_n-\delta) \right]^2}{\sum_n \sin^2 \omega(t_n-\delta)} \right\}$$

Spectral power as a function of angular frequency $\omega = 2\pi f > 0$ with $f \in B_0 = \left[-\frac{1}{2LT}, \frac{1}{2LT}\right]$. where $\bar{x}$ and $\sigma^2$ represent the mean and variance of the samples.
The estimated PSD obtained using Lomb-Scargle method is compared with a threshold $\eta$ in order to get the spectral support $\mathcal{F} = \bigcup_{i=1}^{NB} [a_i, b_i]$.

Once the support $\mathcal{F}$ is found, the set $\mathcal{K} = \{k_r\}_{r=1}^q$, where $k_r \in \{0, 1, \ldots, L - 1\}$, can be calculated as follows:

$$\lfloor a_iLT \rfloor \leq k_i \leq \lfloor b_iLT \rfloor \quad (7)$$

where $1 \leq i \leq N$ and $\lfloor \cdot \rfloor$ is the floor function.
Performance of Lomb-Scargle Method
Once all the $k_i$ are calculated for each band, the set of spectral indexes is given by

$$\mathcal{K} = \bigcup_{i=1}^{N_B} \{k_i\}$$  \hspace{1cm} (8)

- The set $\mathcal{K}$, thus, is sent to the Spectrum Changing Detector block.
- In our proposed DSB sampler, the threshold, $\eta$, is the only information assumed to be available about the input signal [Aziz et al., 2013].
Spectrum Changing Detector block

- Calculating the number of active cells ($N_{\mathcal{K}}$)
  - $N_{\mathcal{K}} = \overline{N}_{\mathcal{K}}$
    - No
    - $\mathcal{K} = \overline{\mathcal{K}}$
      - No
        - Do nothing
      - Yes
        - Updating the spectral support $\mathcal{K}_B$
    - Yes
  - Updating the Value of $\overline{N}_{\mathcal{K}}$

- Statistical study of the spectrum changing instants and sending the new $\overline{\mathcal{K}}$ to OASRS block

- Probability Density

- Piloting switch 1

- Switch 1
Optimal Average Sampling Rate Search Block (OASRS)
Optimal Sampling Pattern

Algorithm 1 SFS algorithm

\textbf{Require:} $L, T, K$

\textbf{Ensure:} $C$

\begin{align*}
& C \leftarrow \emptyset \\
& p \leftarrow |K| \\
& C_s \leftarrow L \\
& i \leftarrow 0 \\
& \textbf{while } i < p \textbf{ do} \\
& \quad \textbf{for } j = 1 \textbf{ to } |C_s| \textbf{ do} \\
& \quad \quad C_{opt} \leftarrow \arg \min[\text{cond}(A_{C \cup C_s(j)}(K))] \\
& \quad \textbf{end for} \\
& \quad C \leftarrow C \cup C_{opt} \\
& \quad C_s \leftarrow C_s - \{C_{opt}\} \\
& \quad i \leftarrow i + 1 \\
& \textbf{end while}
\end{align*}
With $L$, $T$ and $K$ known, SFS algorithm searches for an optimal sampling pattern $C$ which in turn minimizes the reconstruction error. Finally, $C$ is used to compute the elements of the set $\mathcal{T}$. Thus, for a given $L$, the non-uniform sampler operating at an optimal average rate depends only on the number of active band. As a result, the average sampling rate can be written as

$$\bar{f} = \frac{p}{LT} = \frac{|\mathcal{K}|}{LT}$$

(9)
We consider a multiband signal with $N$ bands with a maximum bandwidth of 20MHz.

- 16 QAM modulation symbols are used that are corrupted by the additive white Gaussian noise.
- The wideband of interest is in the range of $\mathcal{B} = [-300, 300]$MHz i.e. $f_{nyq} = 600$MHz.

We assume that the MC sampler has perfect knowledge of the incoming signal while on the other hand, our proposed AMuCoS sampler operates in blind mode and therefore has no information regarding the $\mathcal{F}$ and $N$. 
Numerical results

MC Sampler have an optimal reconstruction ($RMSE = 0.7\%$) for $L = 128$, $p = 33$ and

$$C = \{1, 2, 3, 7, 20, 22, 24, 26, 28, 40, \ldots, 85, 89, 106, 107, 108, 111, 112, 113, 127, 128\}$$
MC Sampler have an optimal reconstruction ($RMSE = 0.7\%$) for SP1 with $L = 128$, $p = 33$ and $\mathcal{C} = \{1, 2, 3, 7, 20, 22, 24, 26, 28, 40, \ldots, 85, 89, 106, 107, 108, 111, 112, 113, 127, 128\}$
Conclusions

- We proposed a new intelligent sampling system for cognitive radio. To ensure optimal reconstruction with a small number of samples, the AMuCoS adapts its parameters according to the input signal.

- We have shown that the average sampling rate depends on the number of bands contained in the signal.

- Its performance has been compared to that of a classical Multi-Coset architecture with p branches. We have shown that our system is significantly more efficient than the conventional MC sampler when the spectrum of signal changes.
Non-uniform spectrum sensing for cognitive radio using sub-nyquist sampling.

In *EUSIPCO, Marrakech- Morocco, 21st European Signal Processing Conference 2013 - Signal Processing for Communications*.

Échantillonnage non uniforme appliqué à la numérisation des signaux radio multistandard.

PhD thesis.

Analysis and design of multirate synchronous sampling schemes for sparse multiband signals.

Least-squares frequency analysis of unequally spaced data.


From theory to practice: Sub-nyquist sampling of sparse wideband analog signals.


Low-cost high-speed pseudo-random bit sequence characterization using nonuniform periodic sampling in the presence of noise.

pages 146 –151.


Non-uniform sampling and reconstruction of multi-band signals and its application in wideband spectrum sensing of cognitive radio.

Scargle, J. D. (1982).

Studies in astronomical time series analysis II. statistical aspects of spectral analysis of unevenly sampled data.


Optimal sub-nyquist nonuniform sampling and reconstruction for multiband signals.