Adaptive Multi-Coset Sampler

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Coexistence of Heterogeneous Wireless Networks
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Outline

1. Introduction
2. Multi-Coset Sampling
3. Adaptive Multi-Coset Sampling
4. Conclusions

Generality

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- Moreover, the current trends in wireless technology have increased the complexity of the receiver, more specifically its Analog to Digital Converter (ADC), due to the nature of broadband signals generated by certain applications, including communication in ultra wideband.
- To sample a wideband signal with Nyquist rate will require a lot of effort and poses a major implementation chanllenge.

Sub-Nyquist Sampling

- [Mishali and Eldar, 2010] proposed for sparse multi-band signal, a sub-Nyquist sampler called Modulated Wideband Converter (MWC). MWC consists of several stages and each stage uses a different mixing function followed by a low pass filter and a low uniform sampling rate. This sampling technique shows that perfect reconstruction is possible when the band locations are known.

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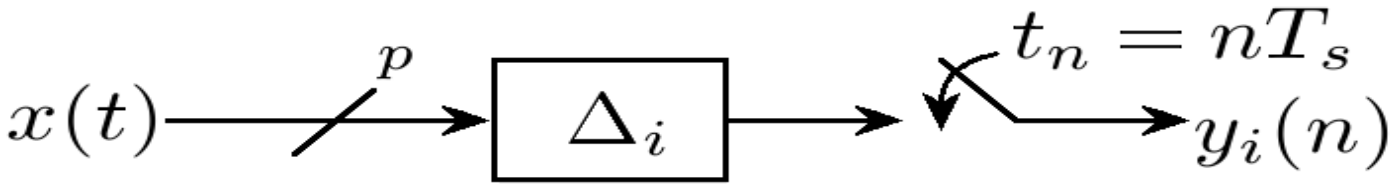
- Multi-Coset (MC) sampling proposed in [Venkataramani and Bresler, 2001] is an effective way to reduce the frequency sampling for multi-band signals whose frequency support is a finite union of intervals.

Multi-Coset sampling

Over the recent years multi-coset sampling has gained fair popularity and several methods of implementing the MC sampling have been proposed.

- The most famous architecture is composed of several parallel branches, each with a time shift followed by a uniform sampler operating at a sampling rate lower than the Nyquist rate



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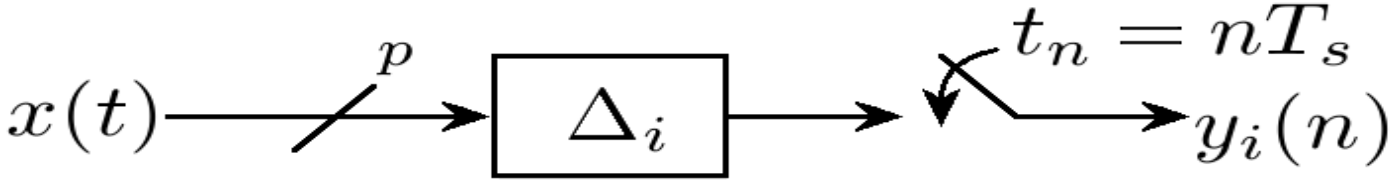

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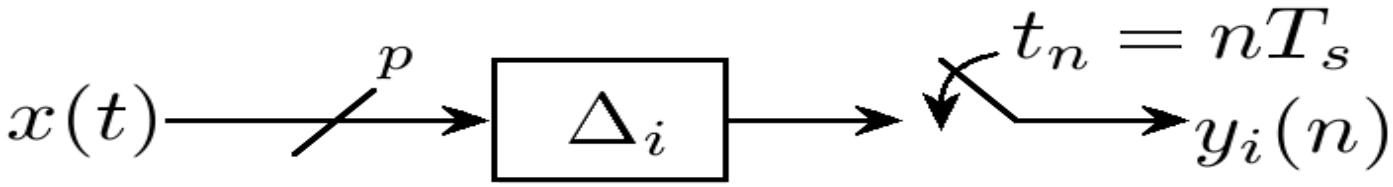


- [Domınguez-Jiménez and González-Prelcic, 2012] uses uniform samplers operating at different rates and is known as the Synchronous Mutlirate Sampling.
- The Dual-Sampling architecture is presented for multi-coset sampling by [Moon et al., 2012]. It is basically a subset of the Synchronous Mutlirate Sampling and uses only two uniform samplers.

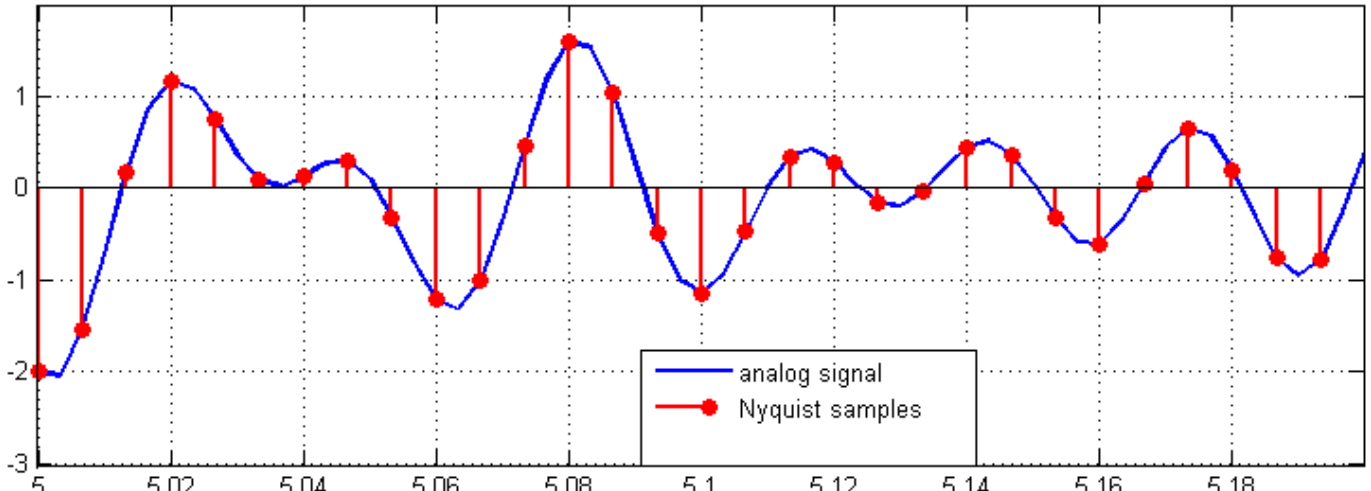
Multi-Coset sampling in time domain

MC sampling is a periodic non-uniform sampling technique which samples a signal at a rate lower than the Nyquist rate [Venkataramani and Bresler, 2001] [Rashidi Avendi, 2010].

Explanation :

- The analog signal $x(t)$ is sampled at Nyquist rate

Example:



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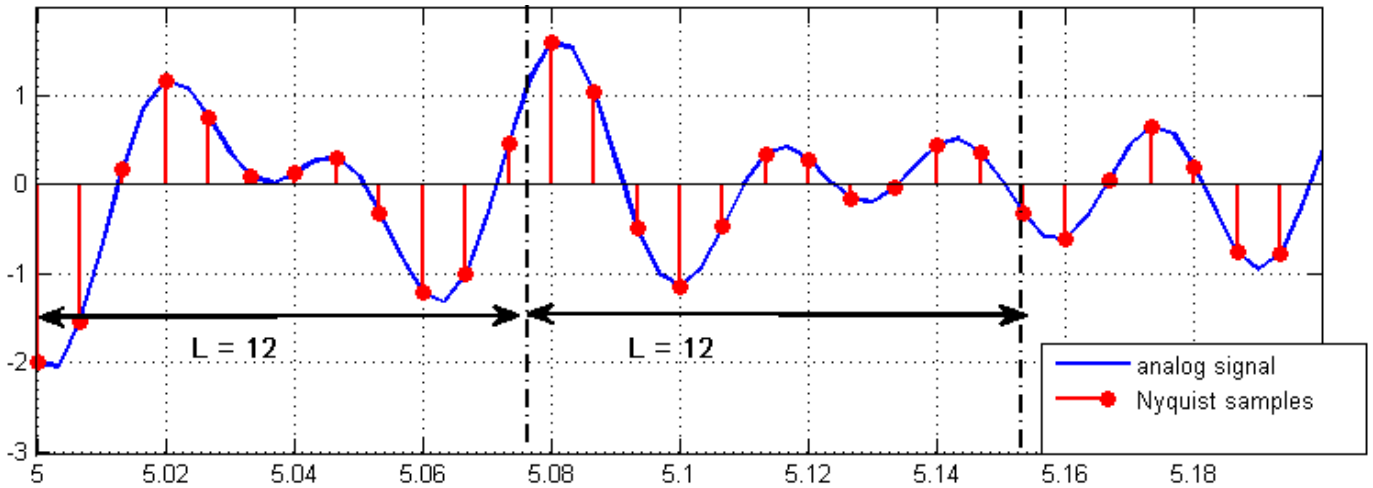
- Divide the Nyquist grid into successive segments of L samples each

Example: $L = 12$,



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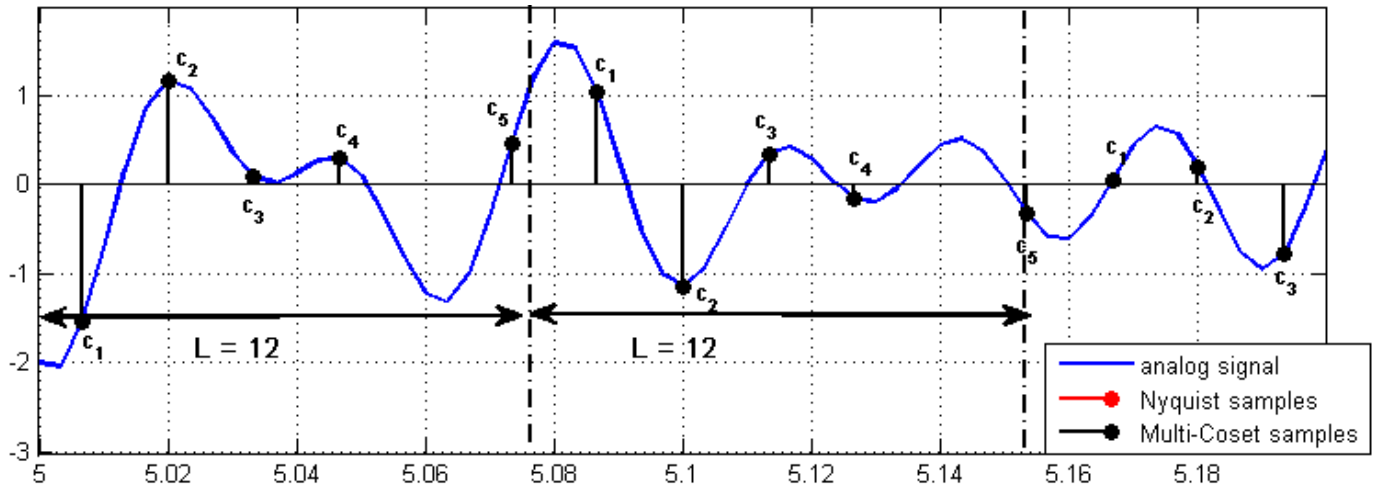
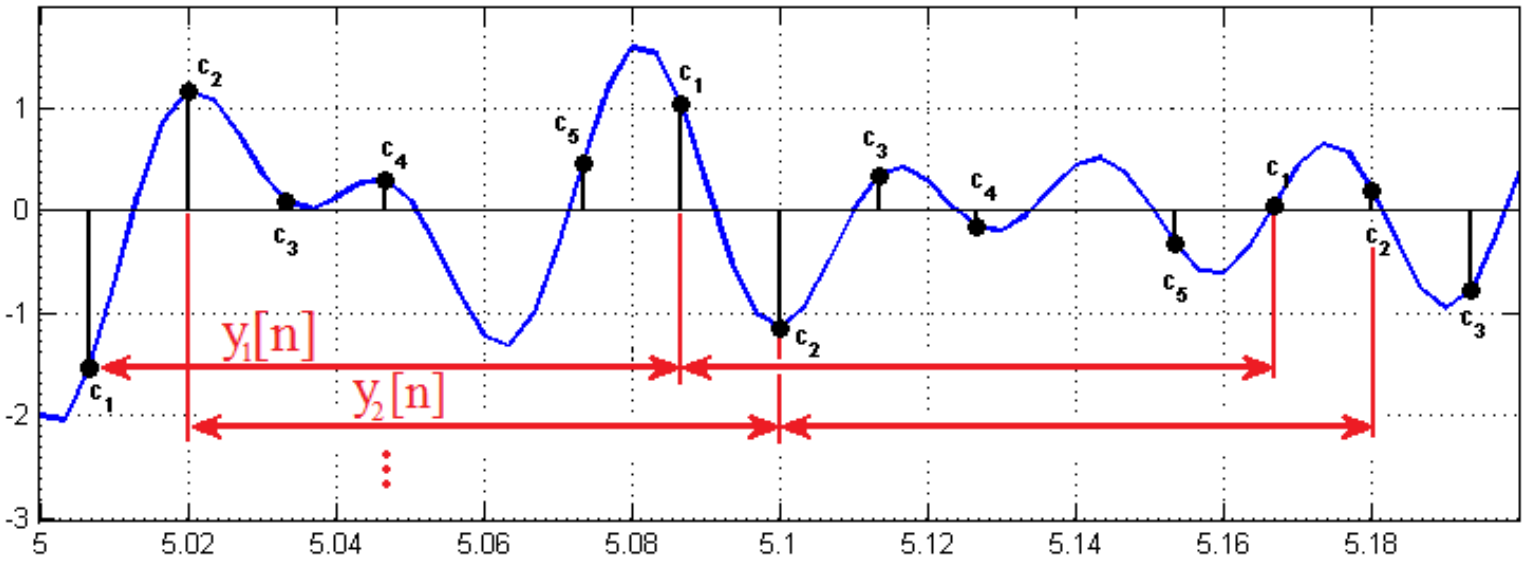
Explanation :

- The analog signal $x(t)$ is sampled at Nyquist rate
- Divide the Nyquist grid into successive segments of L samples each
- In each segment only p samples out of L are kept. Which p samples ? Described by the set $\mathcal{C}$

Example: $L = 12$, $p = 5$, $\mathcal{C} = \{1, 5, 7, 9, 11\}$



Le MC dans le domaine frequentiel (1)



The Fourier transform, $X_i(e^{j2\pi fT})$ of the sampled sequence $y_i[n]$ is related the Fourier transform, $X(f)$, of the unknown signal $x(t)$ by the following equation [Rashidi Avendi, 2010]:

$$\mathbf{y}(f) = \mathbf{A}_{\mathcal{C}}\mathbf{s}(f), f \in \mathcal{B}_0 = [-\frac{1}{2LT}, \frac{1}{2LT}], \quad (1)$$

$\mathbf{y}(f)$ is a vector of size $p \times 1$ whose i^{th} element is given by :

$$\mathbf{y}_i(f) = X_i(e^{j2\pi fT}), f \in \mathcal{B}_0, 1 \leq i \leq p \quad (2)$$

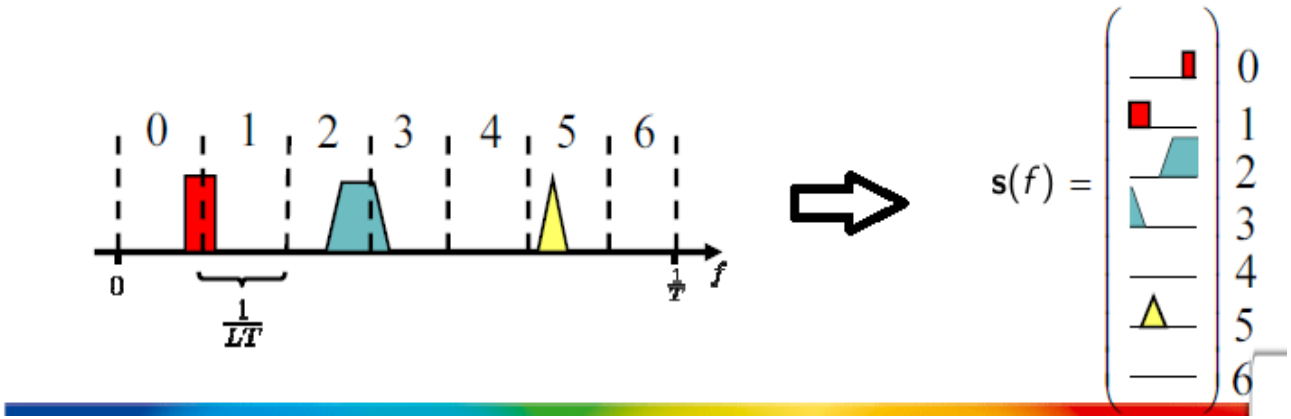
Le MC dans le domaine frequentiel (2)

$\mathbf{A}_{\mathcal{C}}$ is a matrix of size $p \times L$ whose $(i,l)^{th}$ element is given by :

$$[\mathbf{A}_{\mathcal{C}}]_{il} = \frac{1}{LT} exp(\frac{j2\pi l c_i}{L}), 1 \leq i \leq p, 0 \leq l \leq L-1 \qquad (3)$$

$\mathbf{s}(f)$ represents the unknown vector of size $L \times 1$ with l^{th} element given by :

$$\mathbf{s}_l(f) = X(f + \frac{l}{LT}), f \in \mathcal{B}_0, 0 \leq l \leq L-1 \qquad (4)$$



Actives cells $\mathcal{K} = \{0, 1, 2, 3, 5\}$

Multi-Coset reconstruction

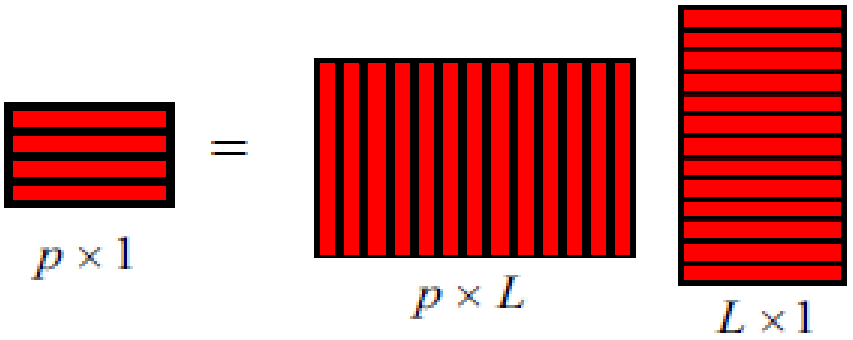
matrix form, under-determined system

$$y(f) = A_c \; S(f)$$

MC samples Fourier transform

sub-matrix of Fourier transform

Nyquist samples Fourier transform



Multi-Coset reconstruction

wholes detection

$$y(f) = A_c \; S(f)$$

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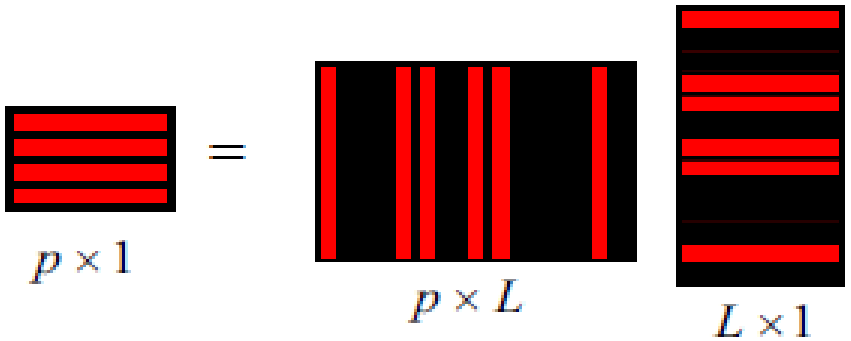


Multi-Coset reconstruction

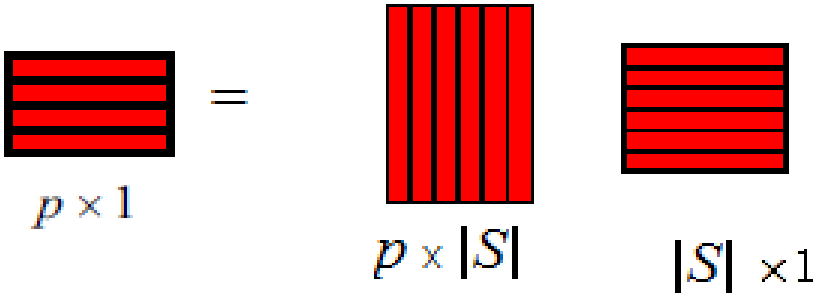
resolvable system

$$y(f) = A_c \; \mathbf{z}(f)$$

MC samples Fourier transform

sub-matrix of Fourier transform

Nyquist samples Fourier transform without wholes



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It is quite evident that once the sampling parameters (such as p) are selected, architecture of the MC sampler will remain unchanged irrespective of the input signal characteristics.
Furthermore Optimal reconstruction that are proposed assume that the number of bands and the maximum bandwidth, a band can have, are known.

Adaptive Multi-Coset Sampler (AMuCoS)

- We present a new sampler, that not only adapts to the changes in the input signal but is also remotely reconfigurable and is, therefore, not constrained by the inflexibility of hardwired circuitry.

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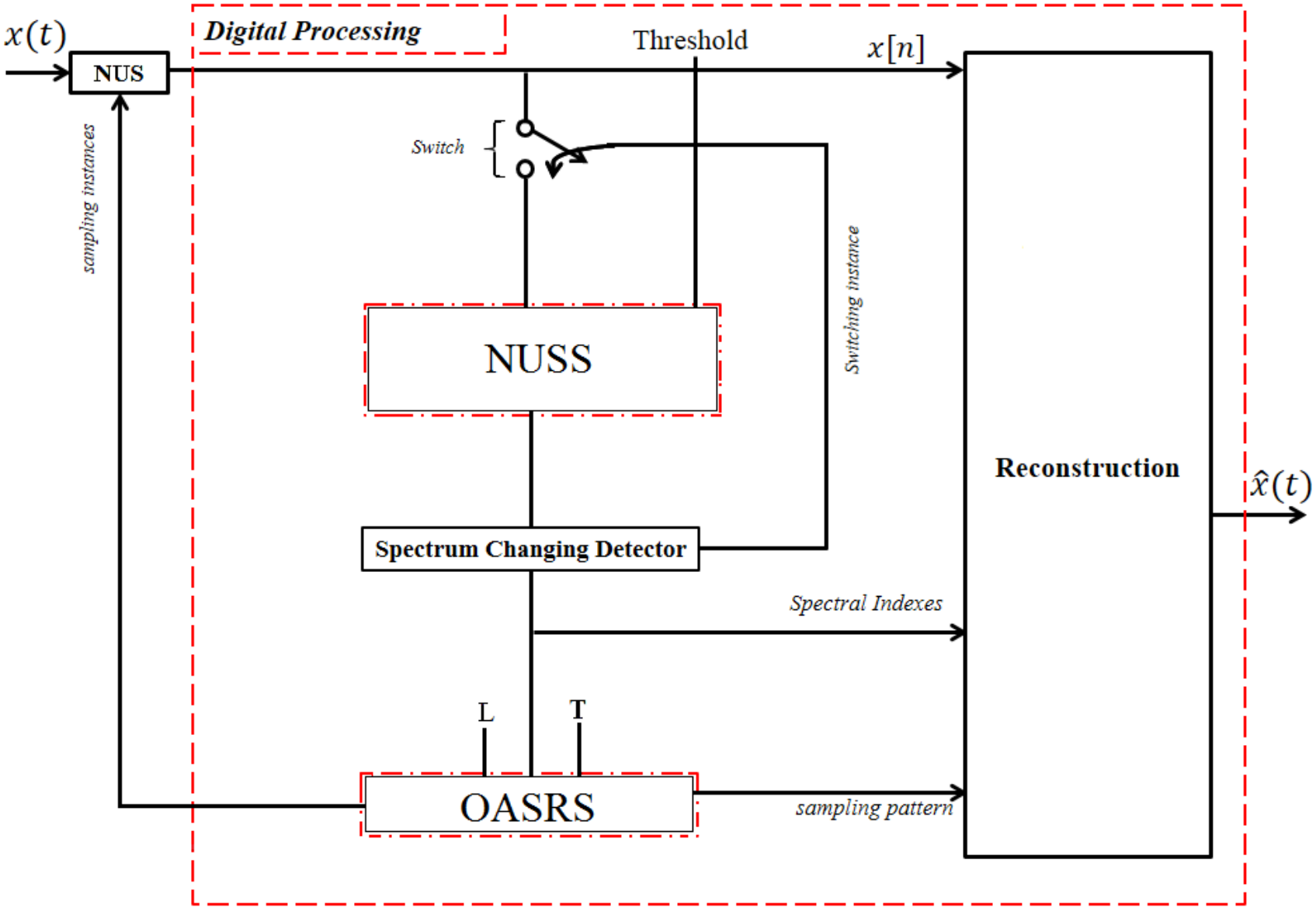
- They call it the *Adaptive Multi-Coset Sampler* or simply the AMuCoS sampler.
- It operates in blind mode, without any knowledge of the input signal's spectral support and the number of bands.

Adaptive Multi-Coset



Non-Uniform Sampler Block (NUS)

We propose to design the NUS of our AMuCoS as a reconfigurable Additive Pseudo-Random Sampler (APRS) in conjunction with MC sampling. In APRS the $\mathcal{N}$ sampling instants are defined as [Ben Romdhane, 2009]:

$$t_m = t_{m-1} + \tau_m = t_0 + \sum_{i=1}^{m} \tau_i, 1 \leq m \leq \mathcal{N}, \tag{5}$$

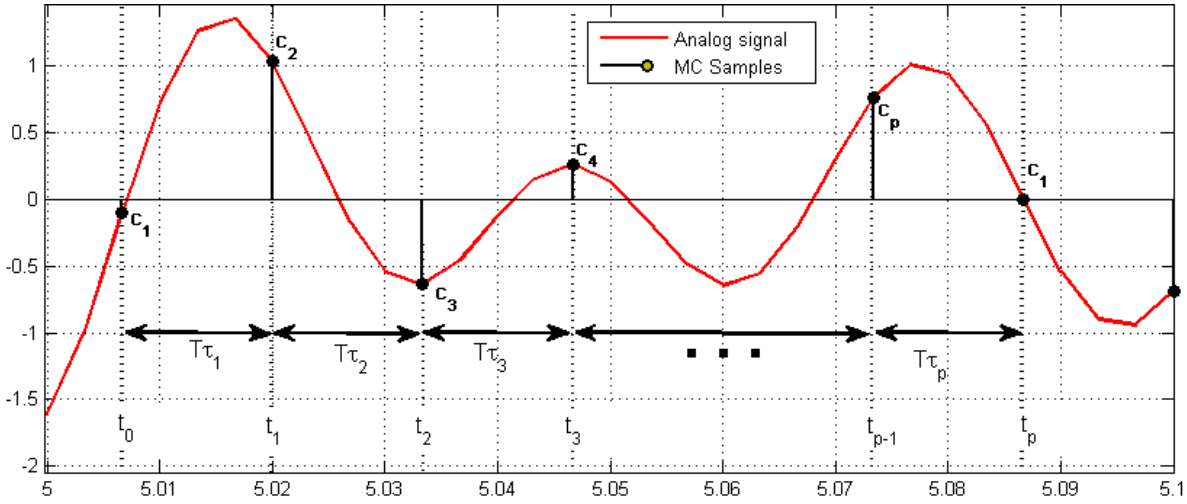
where $E[t_m] = mT$ and $var[t_m] = m\sigma^2$. For $\mathcal{N} \geq 1, \{\alpha_m\}_{m=1}^{\mathcal{N}}$ is a set of i.i.d random variables with density of probability $p_1(\tau)$, mean T and variance σ^2.

Non-Uniform Sampler Block (NUS)

To design an APRS as a MC sampler for a given $\mathcal{C}$ and T. We first defined the set of distances between two sampling instants by $\mathcal{T} = \{\tau_i\}_{i=1}^{p}$ with $\tau_0 = c_1$, $\tau_i = c_{i+1} - c_i$ et $\tau_p = L + c_1 - c_{p-1}$. With $t_0 = T\tau_0$, equation (5) become :

$$t_m = T \sum_{i=0}^{m} \tau_i, \ 0 \leq m \leq p, \ avec \ \tau_i \in \mathbb{N} \tag{6}$$

The set of sampling instants $\{t_n\}_{n\in\mathbb{Z}}$ is non-uniform and periodic like the MC sampling.



Non-Uniform Spectral Sensing Block

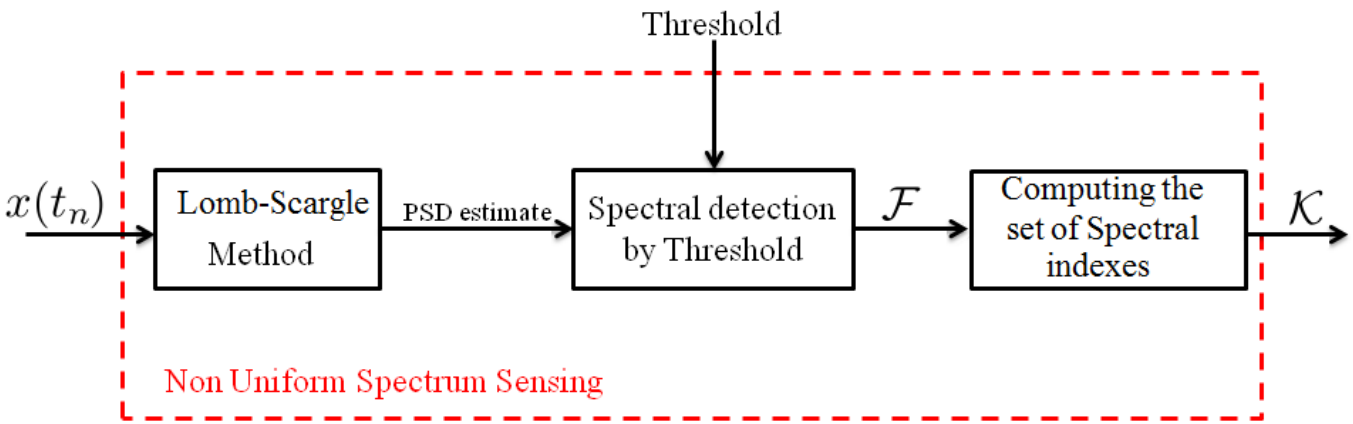
Our System estimates the PSD of the non-uniformly sampled signal by using the Lomb-Scargle method [Lomb, 1976, Scargle, 1982]. Lomb-Scargle method evaluates the samples, only at times t_n that are actually measured.

- Suppose that there are N_s samples $x(t_n)$, $n = 1, ..., N_s$.
- The PSD estimate obtained from Lomb-Scargle Method is defined by :

$$P_{N_s}(\omega) = \frac{1}{2\sigma^2}\left\{\frac{\left[\sum_n (x(t_n)-\bar{x})\cos\omega(t_n-\delta)\right]^2}{\sum_n \cos^2\omega(t_n-\delta)} + \frac{\left[\sum_n (x(t_n)-\bar{x})\sin\omega(t_n-\delta)\right]^2}{\sum_n \sin^2\omega(t_n-\delta)}\right\}$$

Spectral power as a function of angular frequency $\omega = 2\pi f > 0$ with $f \in \mathcal{B}_0 = [-\frac{1}{2LT}, \frac{1}{2LT}]$. where $\bar{x}$ and σ^2 represent the mean and variance of the samples.

Non-Uniform Spectral Sensing Block

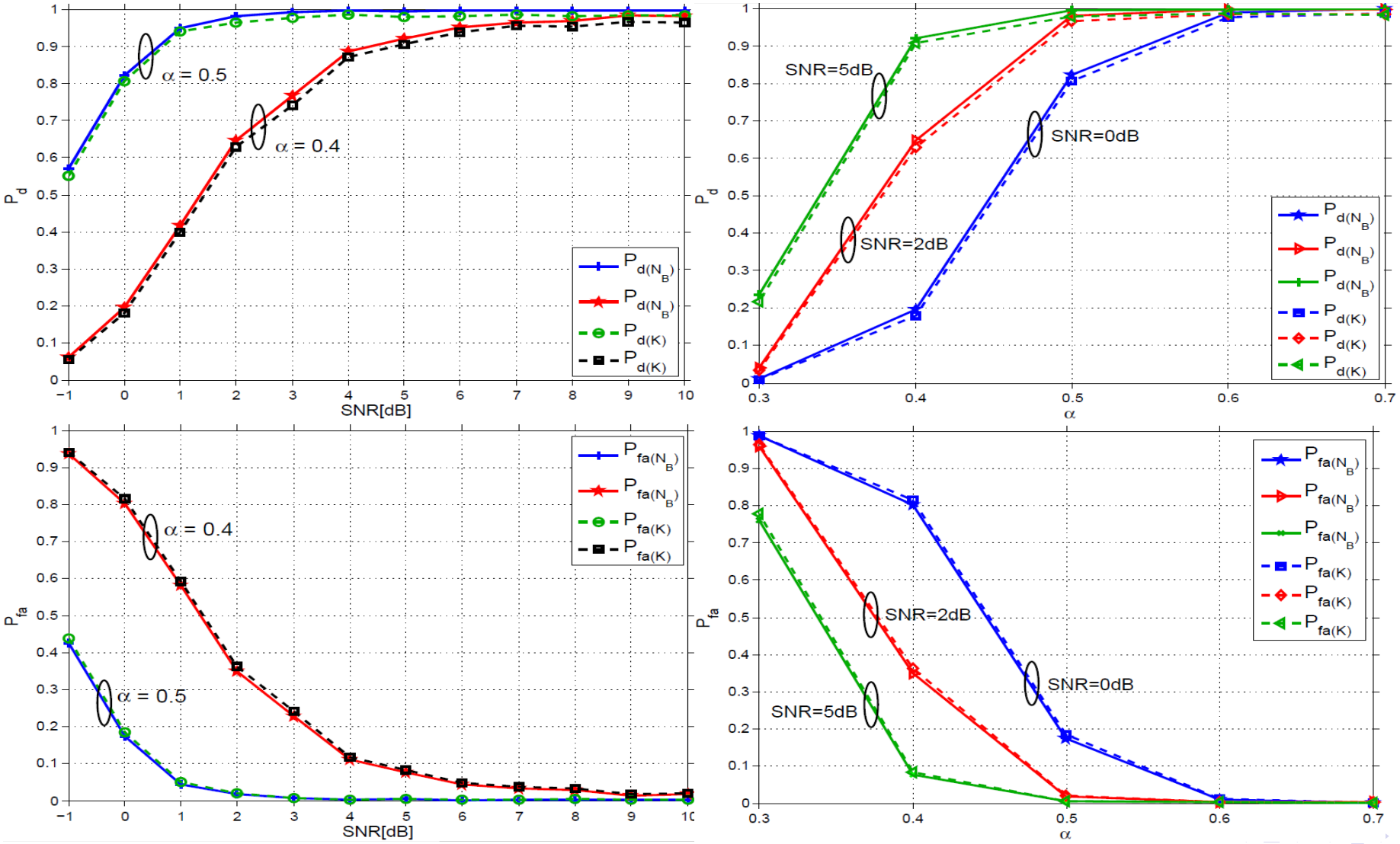


- The estimated PSD obtained using Lomb-Scargle method is compared with a threshold η in order to get the spectral support $\mathcal{F} = \bigcup_{i=1}^{N_B} [a_i, b_i]$.
- Once the support $\mathcal{F}$ is found, the set $\mathcal{K} = \{k_r\}_{r=1}^{q}$, where $k_r \in \{0, 1, ...L-1\}$, can be calculated as follows :

$$\lfloor a_i LT \rfloor \leq k_i \leq \lfloor b_i LT \rfloor \tag{7}$$

where $1 \leq i \leq N$ and $\lfloor * \rfloor$ is the floor function.

Performance of Lomb-Scargle Method



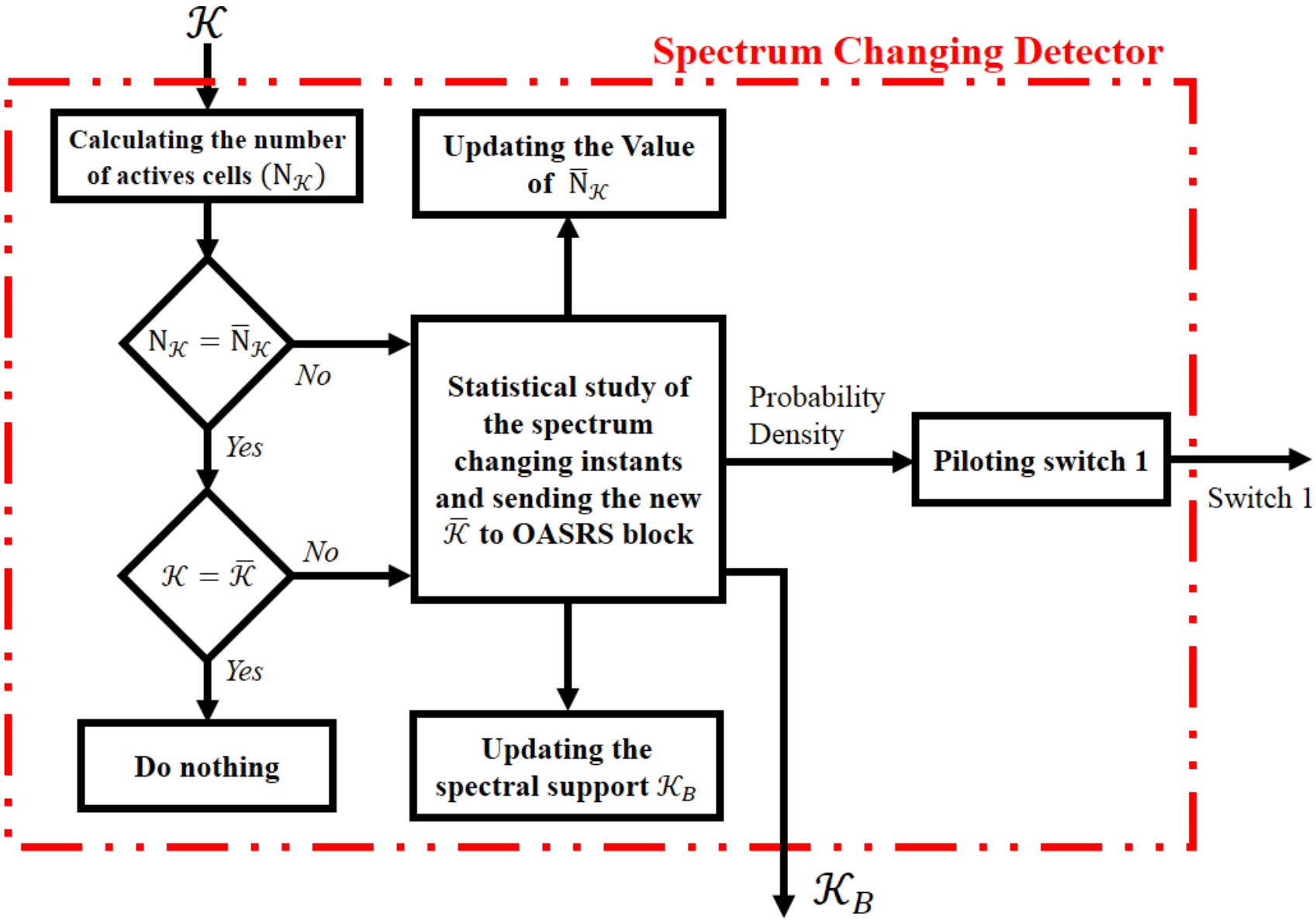
Non-Uniform Spectral Sensing Block

- Once all the k_i are calculated for each band, the set of spectral indexes is given by

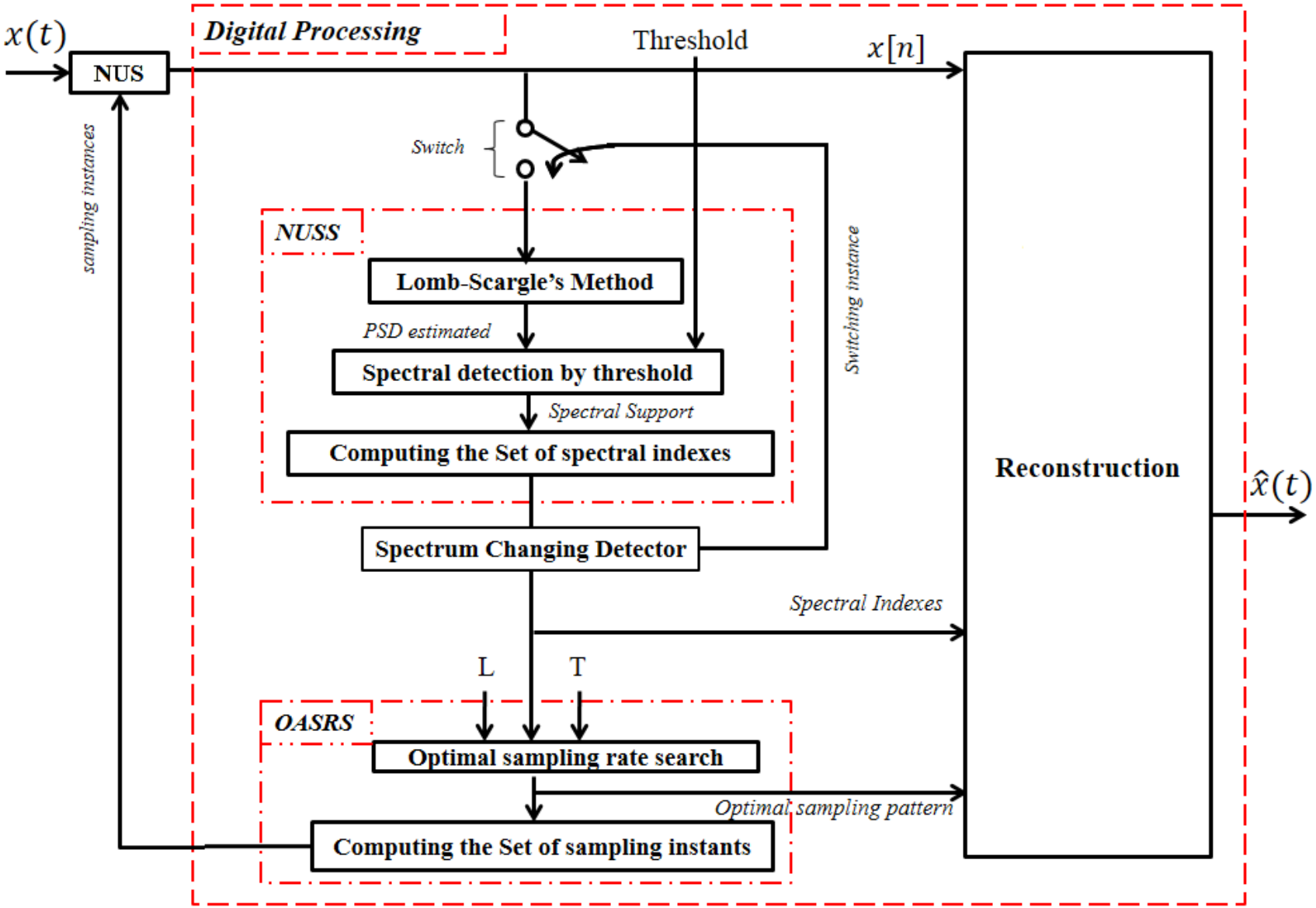
$$\mathcal{K} = \bigcup_{i=1}^{N_B} \{k_i\} \tag{8}$$

- The set $\mathcal{K}$, thus, is sent to the Spectrum Changing Detector block.
- In our proposed DSB sampler, the threshold, η, is the only information assumed to be available about the input signal [Aziz et al., 2013].

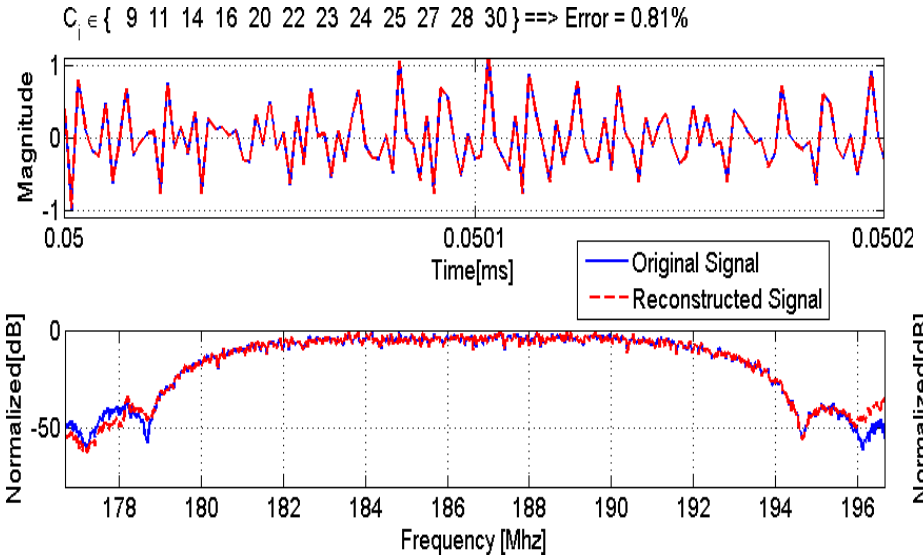
Spectrum Changing Detector block

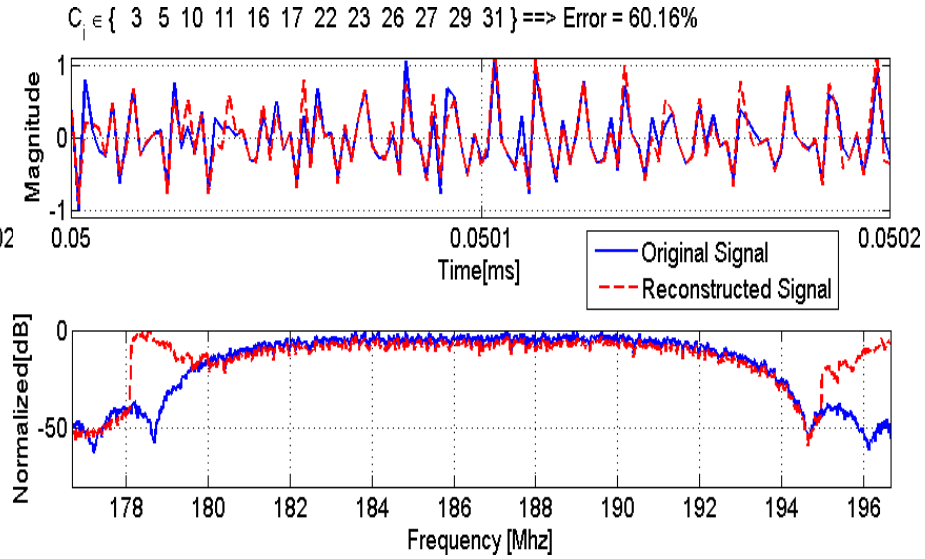


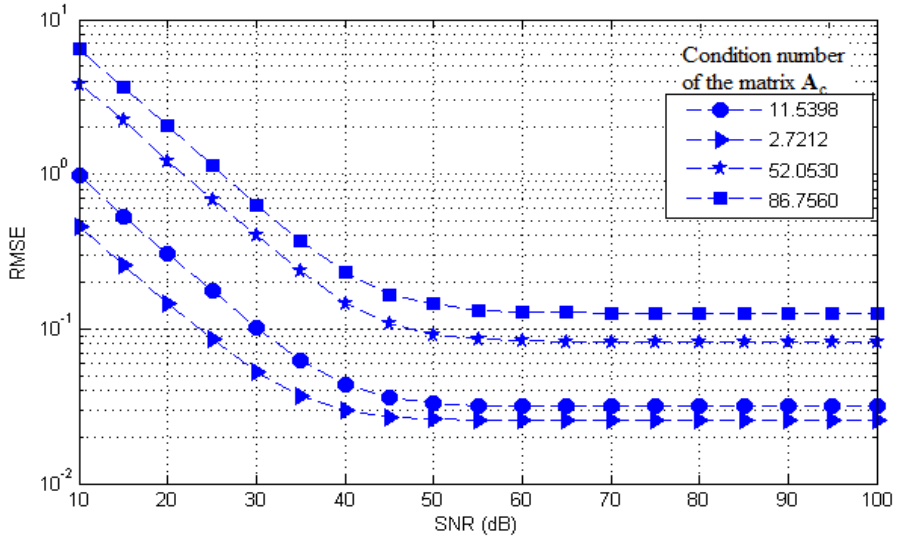
Optimal Average Sampling Rate Search Block (OASRS)



Optimal Sampling Pattern







Algorithm 1 SFS algorithm

Require: $L, T, \mathcal{K}$
Ensure: $\mathcal{C}$
$\mathcal{C} \leftarrow \varnothing$
$p \leftarrow |\mathcal{K}|$
$\mathcal{C}_s \leftarrow \mathbb{L}$
$i \leftarrow 0$
while $i < p$ **do**
  for $j = 1$ to $|\mathcal{C}_s|$ **do**
    $\mathcal{C}_{opt} \leftarrow \arg\min[cond(\mathbf{A}_{\mathcal{C}\cup\mathcal{C}_s(j)}(\mathcal{K}))]$
  end for
  $\mathcal{C} \leftarrow \mathcal{C} \cup \mathcal{C}_{opt}$
  $\mathcal{C}_s \leftarrow \mathcal{C}_s - \{\mathcal{C}_{opt}\}$
  $i \leftarrow i + 1$
end while

Optimal Sampling Pattern

With L, T and $\mathcal{K}$ known, SFS algorithm searchs for an optimal sampling pattern $\mathcal{C}$ which in turn minimizes the reconstruction error. Finally, $\mathcal{C}$ is used to compute the elements of the set $\mathcal{T}$. Thus, for a given L, the non-uniform sampler operating at an optimal average rate depends only on the number of active band. As a result, the average sampling rate can be written as

$$\overline{f} = \frac{p}{LT} = \frac{|\mathcal{K}|}{LT} \tag{9}$$

Numerical results

We consider a multiband signal with N bands with a maximum bandwidth of 20MHz.

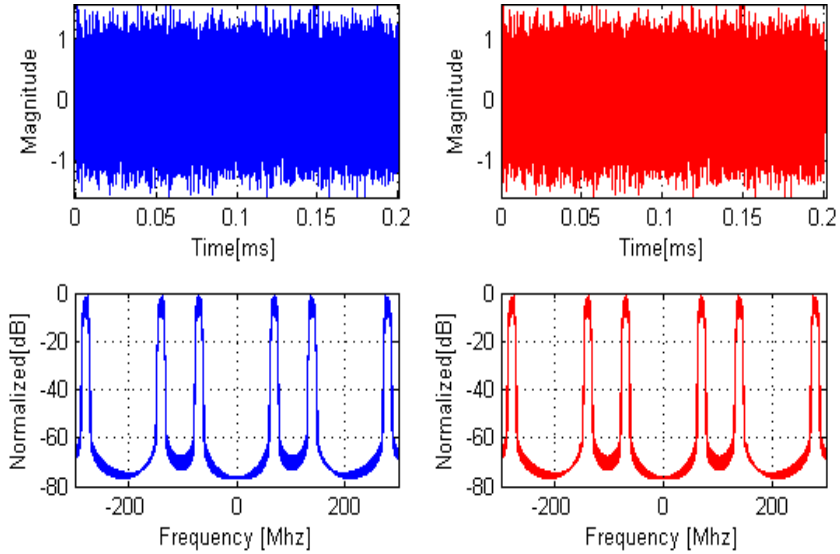
- 16 QAM modulation symbols are used that are corrupted by the additive white Gaussian noise.
- The wideband of interest is in the range of $\mathcal{B} = [-300, 300]$MHz i.e. $f_{nyq} = 600$MHz.

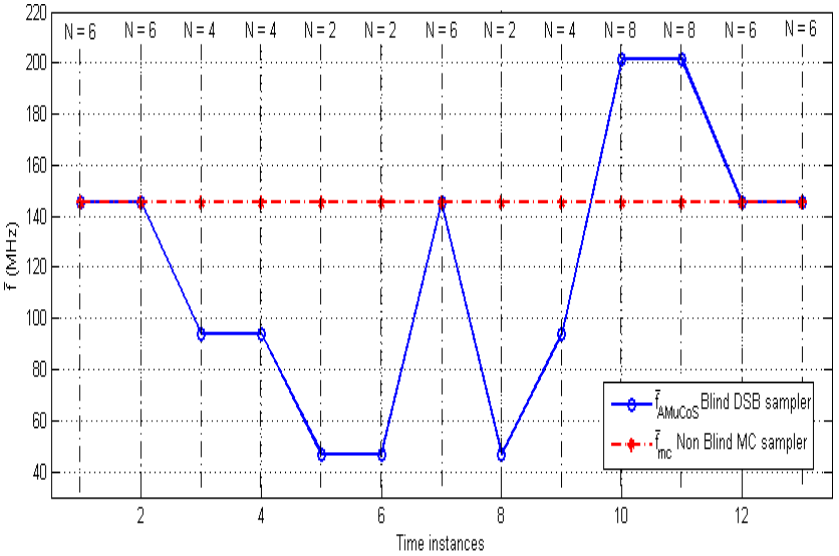
We assume that the MC sampler has perfect knowledge of the incoming signal while on the other hand, our proposed AMuCoS sampler operates in blind mode and therefore has no information regarding the $\mathcal{F}$ and N.

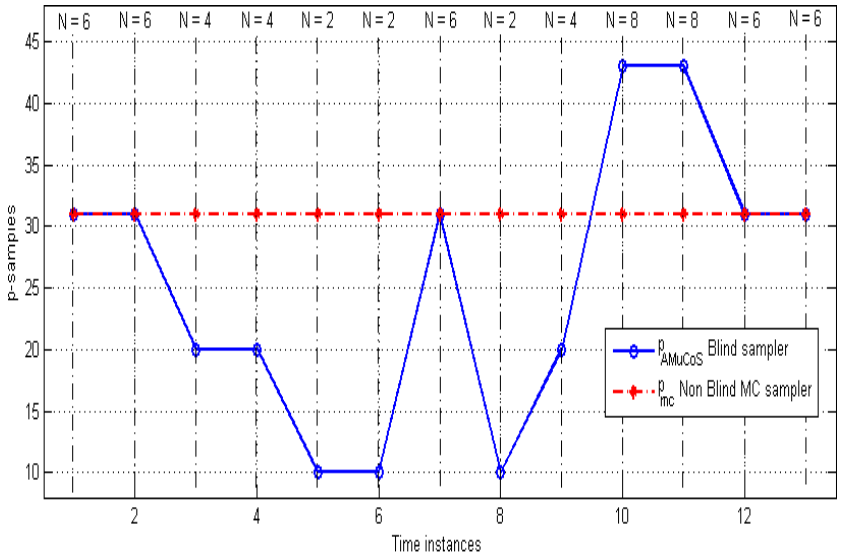
Numerical results

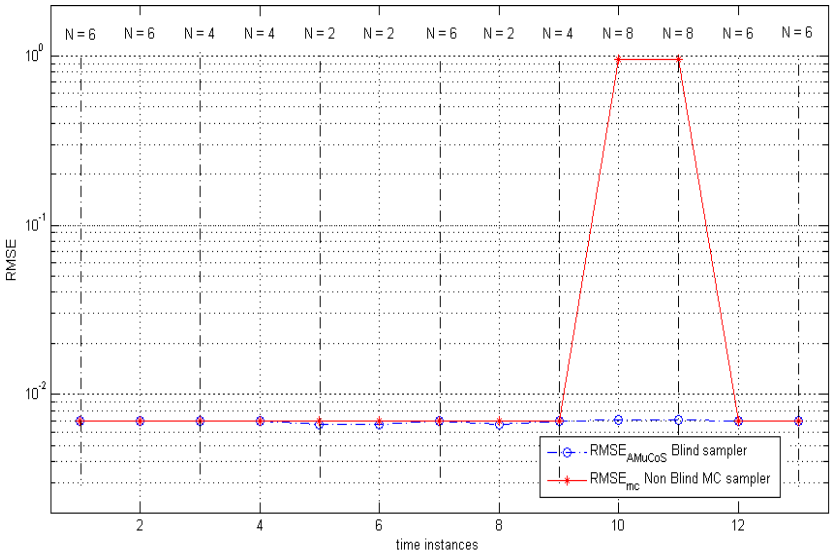
MC Sampler have an optimal reconstruction ($RMSE = 0.7\%$) for $L = 128$, $p = 33$ and

$$\mathcal{C} = \{1, 2, 3, 7, 20, 22, 24, 26, 28, 40, \ldots, 85, 89, 106, 107, 108, 111, 112, 113, 127, 128\}$$









Numerical results

MC Sampler have an optimal reconstruction ($RMSE = 0.7\%$) for SP1 with $L = 128, p = 33$ and $\mathcal{C} =$
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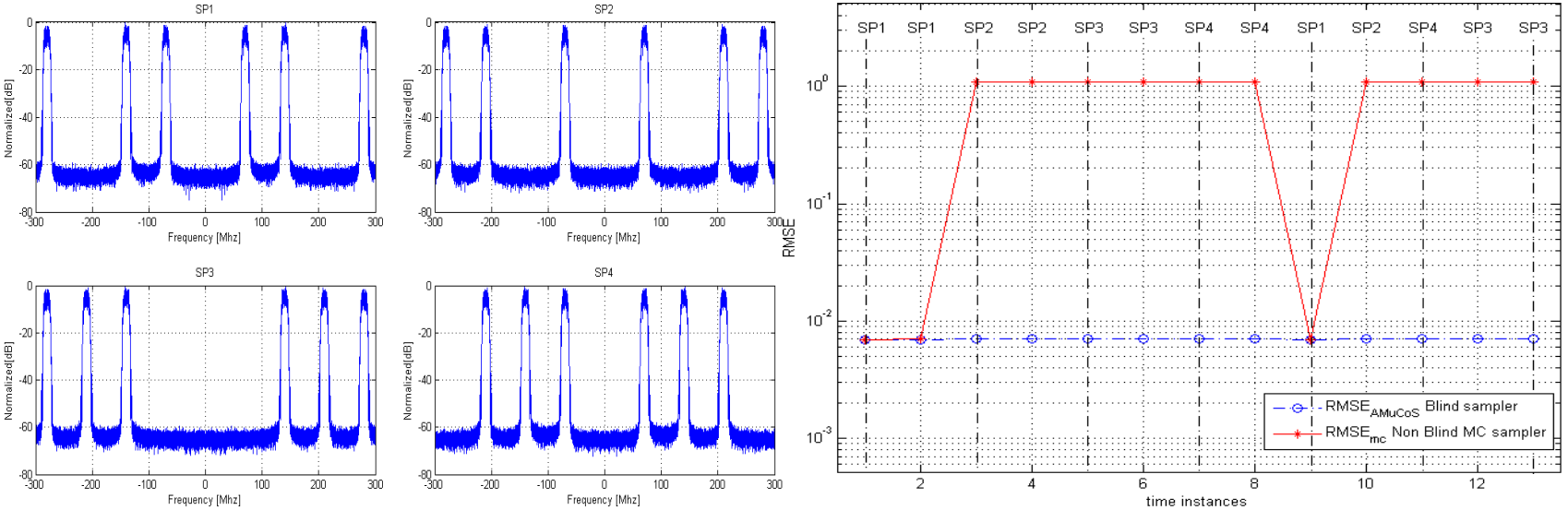

Conclusions

- we proposed a new intelligent sampling system for cognitive radio. To ensure optimal reconstruction with a small number of samples, the AMuCoS adapts its parameters according to the input signal.
- We have shown that the average sampling rate depends on the number of bands contained in the signal.
- Its performance has been compared to that of a classical Multi-Coset architecture with p branches. We have shown that our system is significantly more efficient than the conventional MC sampler when the spectrum of signal changes.

THANKS FOR YOUR ATTENTION

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Aziz, B., Traoré, S., and Le Guennec, D. (2013).

Non-uniform spectrum sensing for cognitive radio using sub-nyquist sampling.

In *EUSIPCO, Marrakech- Morocco, 21st European Signal Processing Conference 2013 - Signal Processing for Communications*.

Ben Romdhane, M. (2009).

Échantillonnage non uniforme appliqué à la numérisation des signaux radio multistandard.

PhD thesis.

Domınguez-Jiménez, M. E. and González-Prelcic, N. (2012).

Analysis and design of multirate synchronous sampling schemes for sparse multiband signals.

Lomb, N. R. (1976).

Least-squares frequency analysis of unequally spaced data.

Astrophysics and Space Science, 39:447–462.

Mishali, M. and Eldar, Y. C. (2010).

From theory to practice: Sub-nyquist sampling of sparse wideband analog signals.

Selected Topics in Signal Processing, IEEE Journal of, 4(2):375–391.

Moon, T., Tzou, N., Wang, X., Choi, H., and Chatterjee, A. (2012).

Low-cost high-speed pseudo-random bit sequence characterization using nonuniform periodic sampling in the presence of noise.

pages 146 –151.

Rashidi Avendi, M. (2010).

Non-uniform sampling and reconstruction of multi-band signals and its application in wideband spectrum sensing of cognitive radio.

Scargle, J. D. (1982).

Studies in astronomical time series analysis II. statistical aspects of spectral analysis of unevenly sampled data.

Astrophysical Journal, 263:835–853.

Venkataramani, R. and Bresler, Y. (2001).

Optimal sub-nyquist nonuniform sampling and reconstruction for multiband signals.

Signal Processing, IEEE Transactions on, 49(10):2301–2313.