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Joint Localization Algorithms for Network Topology Ambiguity Reduction

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Outline

- Introduction
- Problem definition
- Joint Maximum Likelihood (JML)
- Concluding remarks

Introduction

- Transmitter localization is an important aspect in commercial, public safety and military applications of current and future wireless networks
  - *Numerous approaches possible*
- Received Signal Strength (RSS)-based localization is a viable localization solution due to the:
  - Inherent presence of the RSS extraction feature in all radio devices
  - Sufficient precision for a variety of practical applications
- However, there are associated challenges in the process as the operating environment of wireless networks is very hostile + the radio environmental information (e.g. wireless channel model parameters) and network configuration information (sensor positions) might be unreliable or absent
- This presentation analyzes the network topology ambiguity problem and proposes novel localization algorithms that improve the transmitter localization performance while reducing the network topology ambiguity

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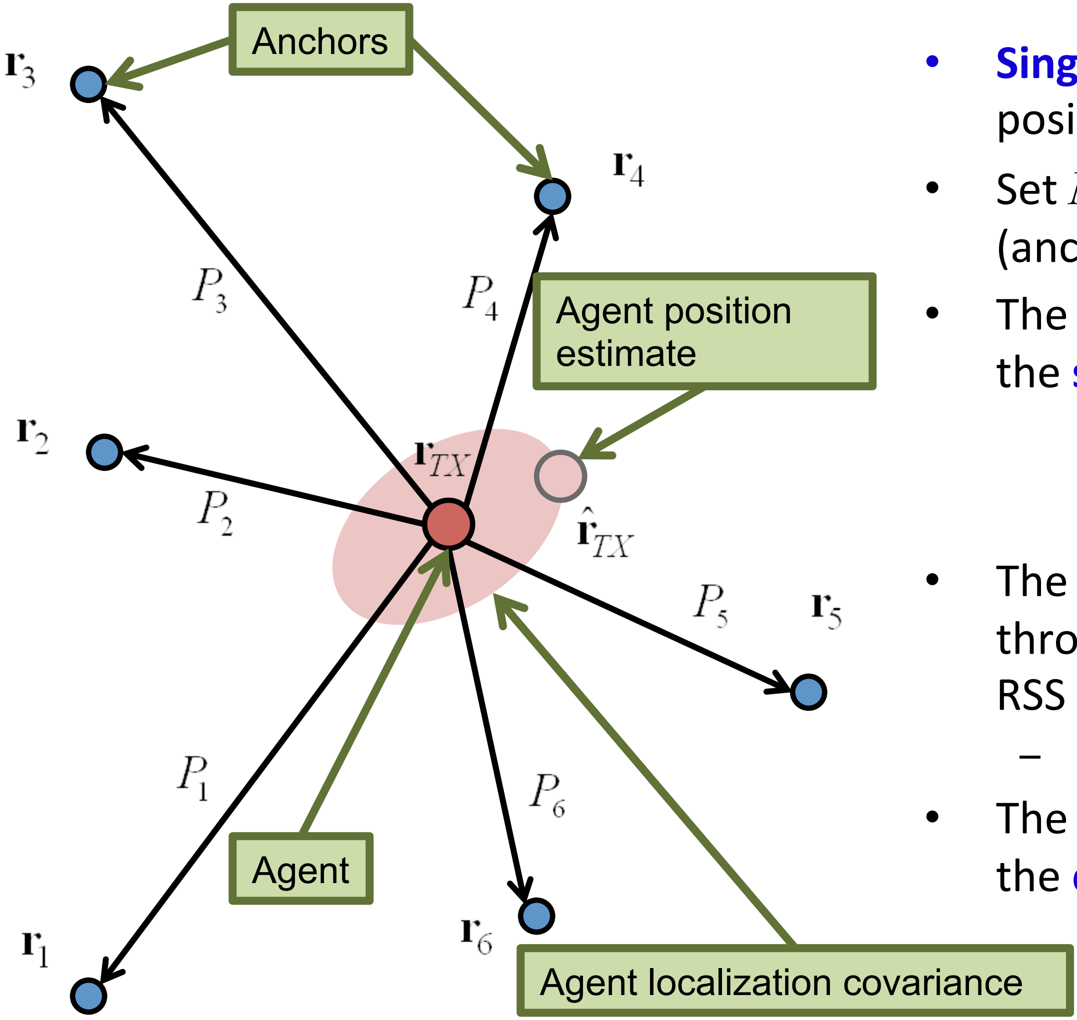
General network setup

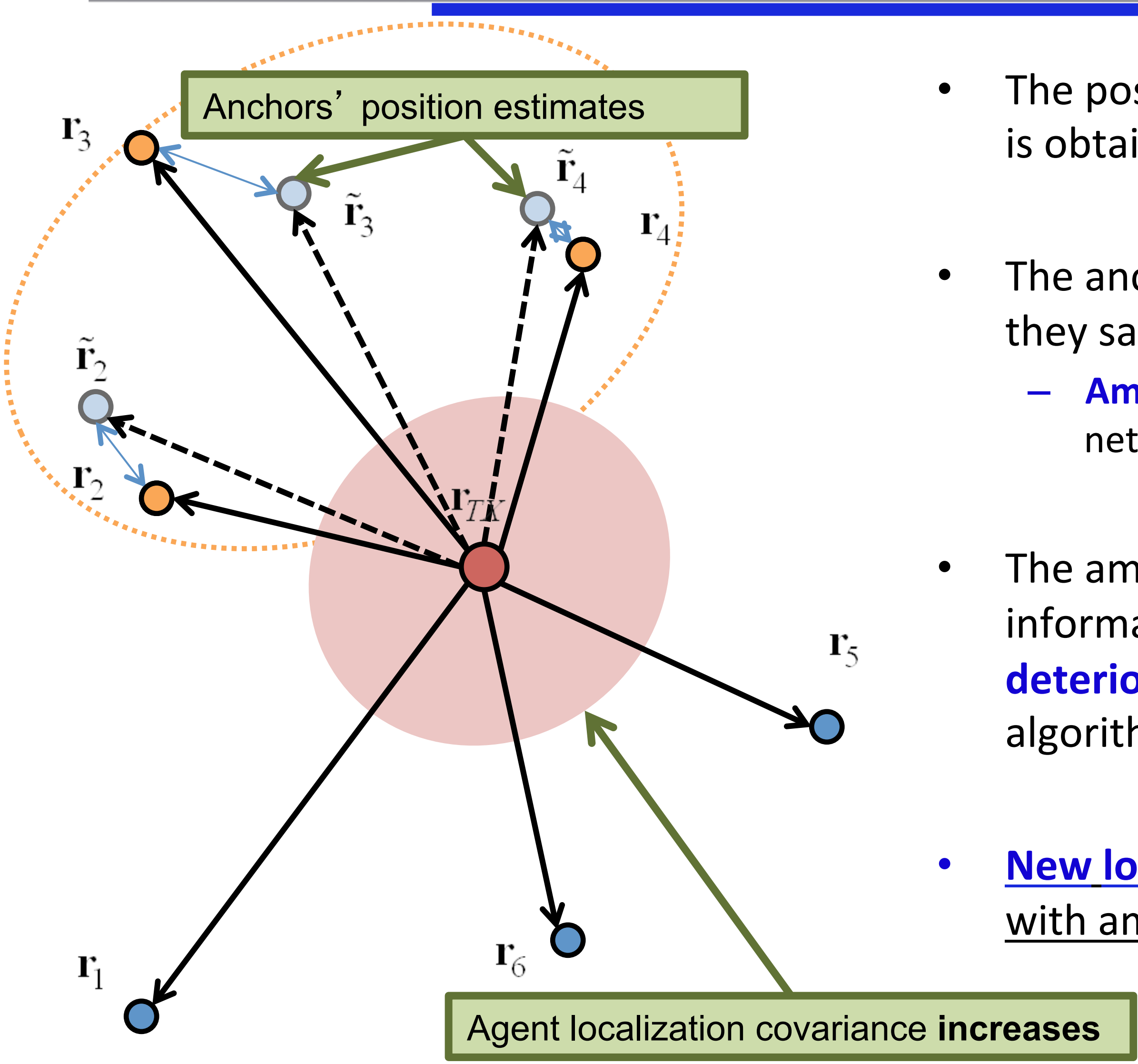


- **Single transmitter** (agent) with unknown position $\mathbf{r}_{TX}$
- Set N, $|N|=n$ of **measuring sensors** (anchors) with known positions $\mathbf{r}_i;\ i \in N$
- The anchors measure the **RSS** $P_i;\ i \in N$ of the **signal** broadcasted by the transmitter
- The transmitter position $\hat{\mathbf{r}}_{TX}$ estimated through (unbiased) **estimation** using the RSS observations
  - Using appropriate **path loss model***
- The **localization error** quantified through the **covariance matrix**

$$\mathbf{C}_{\hat{\mathbf{r}}_{TX}} = \mathrm{E}\left[\left(\hat{\mathbf{r}}_{TX} - \mathbf{r}_{TX}\right)\left(\hat{\mathbf{r}}_{TX} - \mathbf{r}_{TX}\right)^T\right]$$

** R. K. Martin, and R. Thomas, "Algorithms and bounds for estimating location, directionality, and environmental parameters of primary spectrum users," IEEE Transaction on Wireless Communications 8(11), pp. 5692-5701, November 2009.*

Anchor position ambiguity



- The position information of some anchors is obtained through **previous estimation**
- The anchor position **estimates** are $\tilde{\mathbf{r}}_i$ and they satisfy $\tilde{\mathbf{r}}_i \neq \mathbf{r}_i$
  - **Ambiguous** and often very **unreliable** network topology information
- The ambiguous anchor position information $\tilde{\mathbf{r}}_i$ can cause **severe deterioration** of the localization algorithms' performance
- **New localization algorithms** for scenarios with ambiguous topologies **needed**

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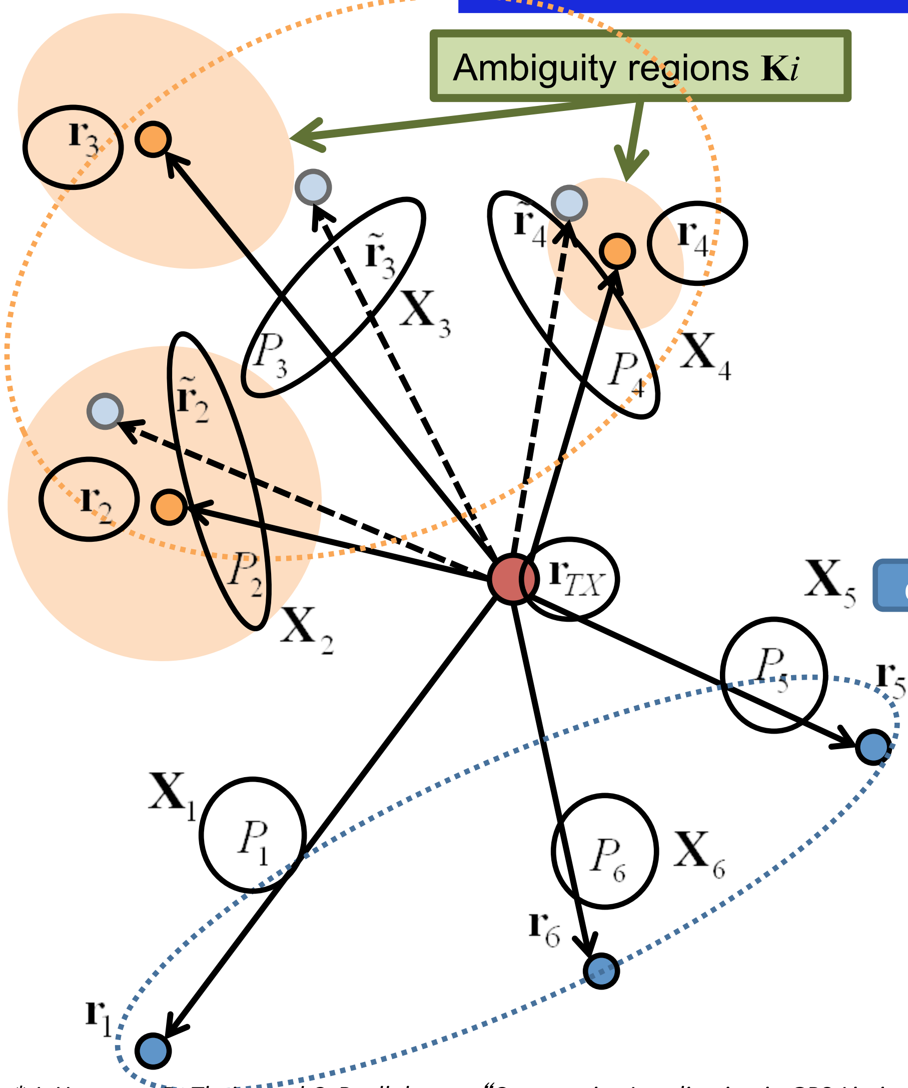
Joint localization algorithms

- The ambiguity problem can be alleviated with localization algorithms that **jointly** estimate the **agent's** and the **anchors' exact positions***
  - **Improve** the **transmitter localization** performance
  - **Reduce** the network topology **ambiguity**
- The joint localization relies on the assumption that the erroneous information about the anchor positions is obtained by some previous estimation
  - Modeled as a **random process** parameterized with respect to the exact anchor positions
- This work presents a **general joint RSS-based localization** framework developed using **non-Bayesian estimation formalism***
  – The unknown **agent's** and ambiguous **anchors**' positions are regarded as **deterministic parameters** → the technically correct estimation approach for **scenarios with imprecise anchor position information**

** M. Angjelicinoski, D. Denkovski, V. Atanasovski, and L. Gavrilovska, "SPEAR: Source Position Estimation for Ambiguity Reduction," IEEE Communication Letters (submitted)*

Joint localization algorithms: Non-Bayesian formalism



- Two subsets of anchors:
  - Subset V, $|V|=v$ -> anchors with **certain** i.e. **precisely known** position (Base Stations or APs)
  - Subset U, $|U|=u$ -> anchors with **ambiguous** positions
- The **data vector** is $\mathbf{X}=(\mathbf{X}_1,\ldots,\mathbf{X}_n)^T \in \mathbb{R}^{n+2u}$

$$\mathbf{X}_i = \begin{cases} (P_i \;\; \tilde{\mathbf{r}}_i)^T & i \in U \\ P_i & i \in V \end{cases}$$

- The **unknown parameters** vector is

deterministic

$$\boldsymbol{\theta} = (\mathbf{r}_{TX},\ldots,\mathbf{r}_i,\ldots)^T_{i\in U} \in \mathbb{R}^{2+2u}$$

- **Assumptions:**
  - Propagation model: ***log-distance path loss model in log-normal uncorrelated shadowing*** $\sigma(\text{in } dB)$

$$P_i \sim \aleph\left(\delta_i(\boldsymbol{\theta}),\sigma^2\right);\; i \in N$$

$$\delta_i(\boldsymbol{\theta}) = P_{TX} - L_0 - 10\gamma\log_{10}\left(\|\mathbf{r}_{TX}-\mathbf{r}_i\|/d_0\right)$$

  - Anchor position estimates: ***i.i.d. Gaussian****

$$\tilde{\mathbf{r}}_i \sim \aleph\left(\mathbf{r}_i,\mathbf{K}_i\right);\; i \in U$$

** J. Hemmes, D. Thain, and C. Poellabauer, "Cooperative Localization in GPS-Limited Urban Environments," First International Conference ADHOCNETS 2009, Ontario, CA, 2009, pp. 422-437.*

Joint Maximum Likelihood

- The main task of the RSS-based joint localization framework is to **estimate $\boldsymbol{\theta}$ based on the information contained in $\mathbf{X}$***

- Employing **Maximum Likelihood** (ML) approach* results in the **Joint Maximum Likelihood** (JML) localization algorithm

$$\hat{\boldsymbol{\theta}}_{JML} = \arg\max_{\boldsymbol{\theta}} \left\{ \ln p(\mathbf{X};\boldsymbol{\theta}) \right\} = \arg\max_{\boldsymbol{\theta}} \left\{ -\frac{1}{\sigma^2} \sum_{i=1}^{n} \left(P_i - \delta_i(\boldsymbol{\theta}) \right)^2 - \sum_{i \in U} \left(\tilde{\mathbf{r}}_i - \mathbf{r}_i \right)^T \mathbf{K}_i^{-1} \left(\tilde{\mathbf{r}}_i - \mathbf{r}_i \right) \right\}$$

where $p(\mathbf{X};\boldsymbol{\theta})$ is **the joint probability density function** of the data vector $\mathbf{X}$

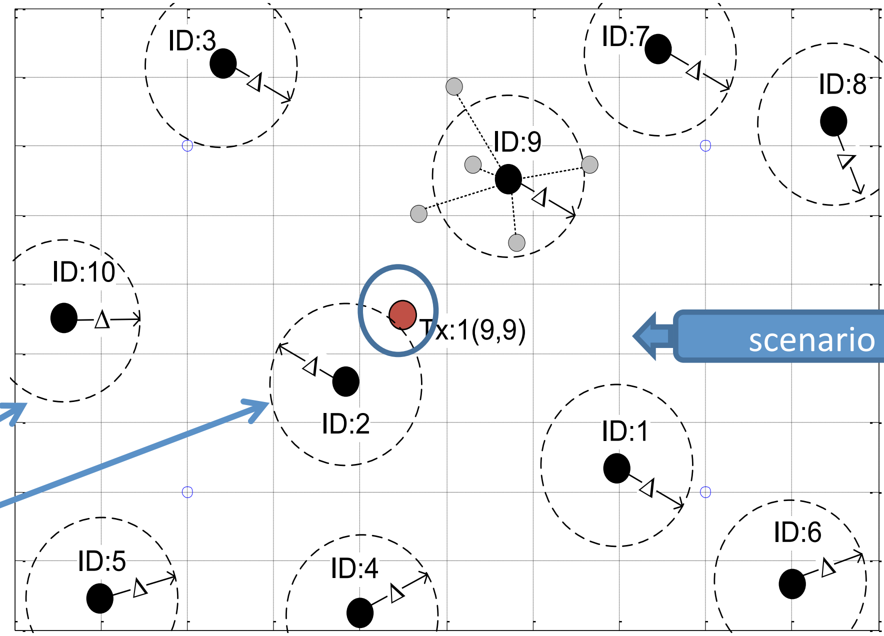
- When $\tilde{\mathbf{r}}_i = \mathbf{r}_i;\ i \in N$, then $\boldsymbol{\theta} = \mathbf{r}_{TX}$ and the JML becomes the **Legacy ML** (LML) localization algorithm

$$\hat{\boldsymbol{\theta}}_{LML} = \arg\min_{\boldsymbol{\theta}} \left\{ \sum_{i=1}^{n} \left(P_i - \delta_i(\boldsymbol{\theta}) \right)^2 \right\}$$

* *S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory, Prentice-Hall, 1998.*

JML performance evaluation (1/4) → transmitter localization

Simulation parameters	
Simulated area	20mX20m
Transmit power	0dBm
Path loss exponent	2.5
Reference distance	1m
Transmitter position	(9,9)m
Shadowing std. deviation	1:1:10 dB
Number of anchors	10
Number of ambiguous anchors	10 (U=N)
Anchor ambiguity std. deviation	1:3:10 m
Monte-Carlo trials (L)	10000



scenario

Circular normal $\mathbf{K}_i = \Delta^2 \mathbf{I}_2;\ i \in U$

$$\hat{\boldsymbol{\theta}}_{JML} = \arg\min_{\boldsymbol{\theta}} \left\{ \sum_{i=1}^{n} \left(\sigma^{-2} \left(P_i - \delta_i(\boldsymbol{\theta}) \right)^2 + \Delta^{-2} \left\| \tilde{\mathbf{r}}_i - \mathbf{r}_i \right\|^2 \right) \right\}$$

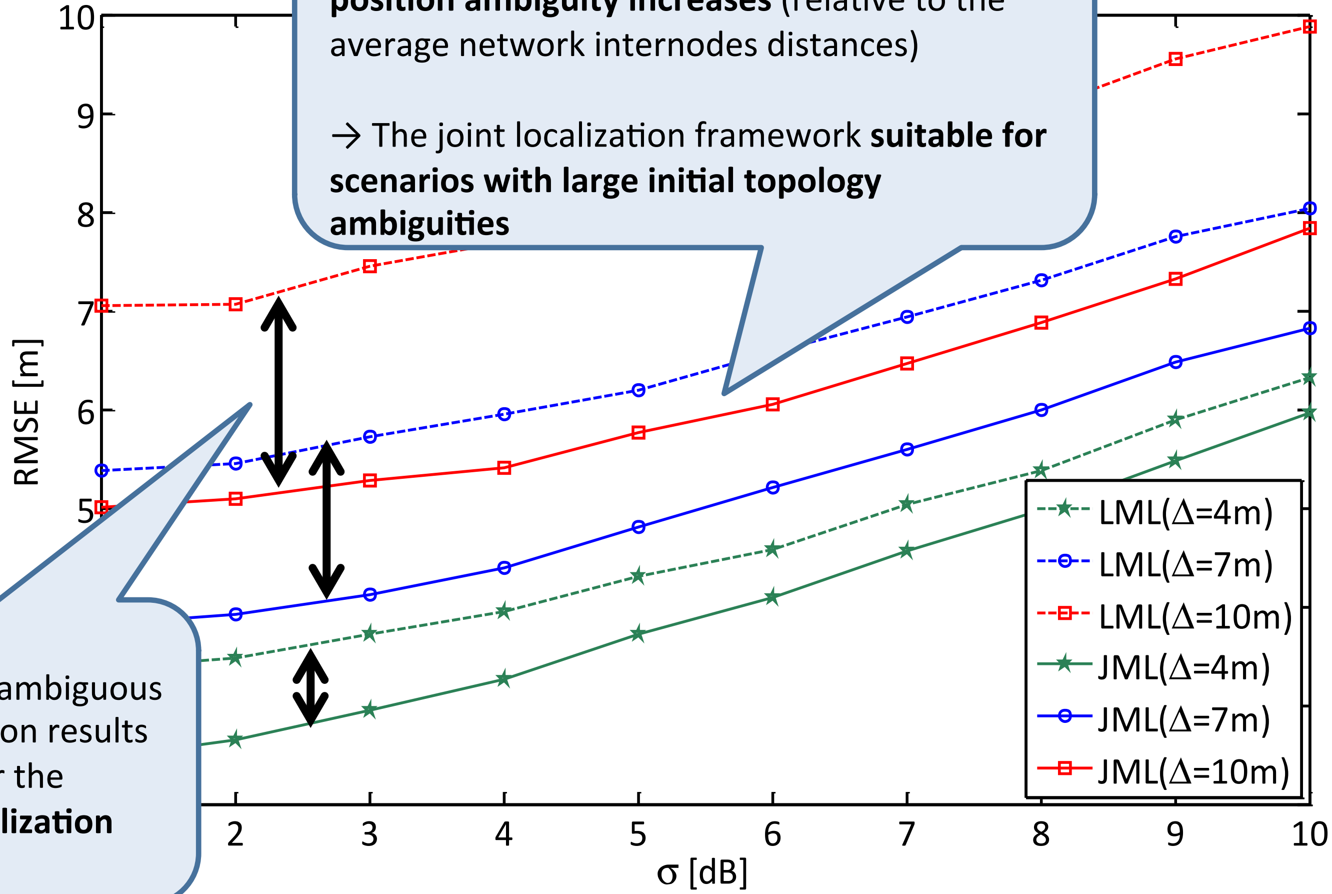
GOAL: Compare the **Root Mean Squared Error** (RMSE) performance of the JML with the performance of the LML in terms of the **transmitter localization capabilities**

$$RMSE\left(\hat{\mathbf{r}}_{TX}; \sigma, \Delta\right) = \sqrt{\frac{1}{L} \sum_{j=1}^{L} \left\| \hat{\mathbf{r}}_{TXj} - \mathbf{r}_{TX} \right\|^2};\ \hat{\mathbf{r}}_{TXj} = \begin{cases} [\hat{\boldsymbol{\theta}}_{JMLj}]_{1:2}\ ; & \mathbf{JML} \\ \hat{\boldsymbol{\theta}}_{LMLj}\ ; & \mathbf{LML} \end{cases}$$

JML performance evaluation (2/4) → transmitter localization

The **RMSE gain increases as** the initial **anchor position ambiguity increases** (relative to the average network internodes distances)

→ The joint localization framework **suitable for scenarios with large initial topology ambiguities**

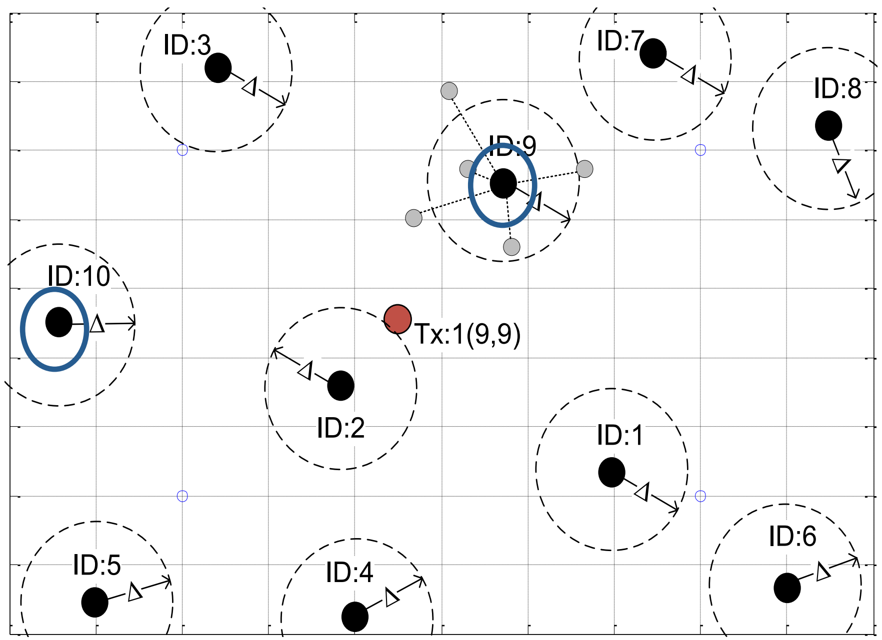


The joint agent/ambiguous anchor localization results in **RMSE gain** for the **transmitter localization**

The JML performs better than the LML → ***reliable transmitter localization is possible in scenarios with ambiguous anchor positions***

JML performance evaluation (3/4) → anchor ambiguity reduction

GOAL: Compare the **Root Mean Squared Error** (RMSE) performance of the JML in terms of **reducing the initial network topology ambiguity**



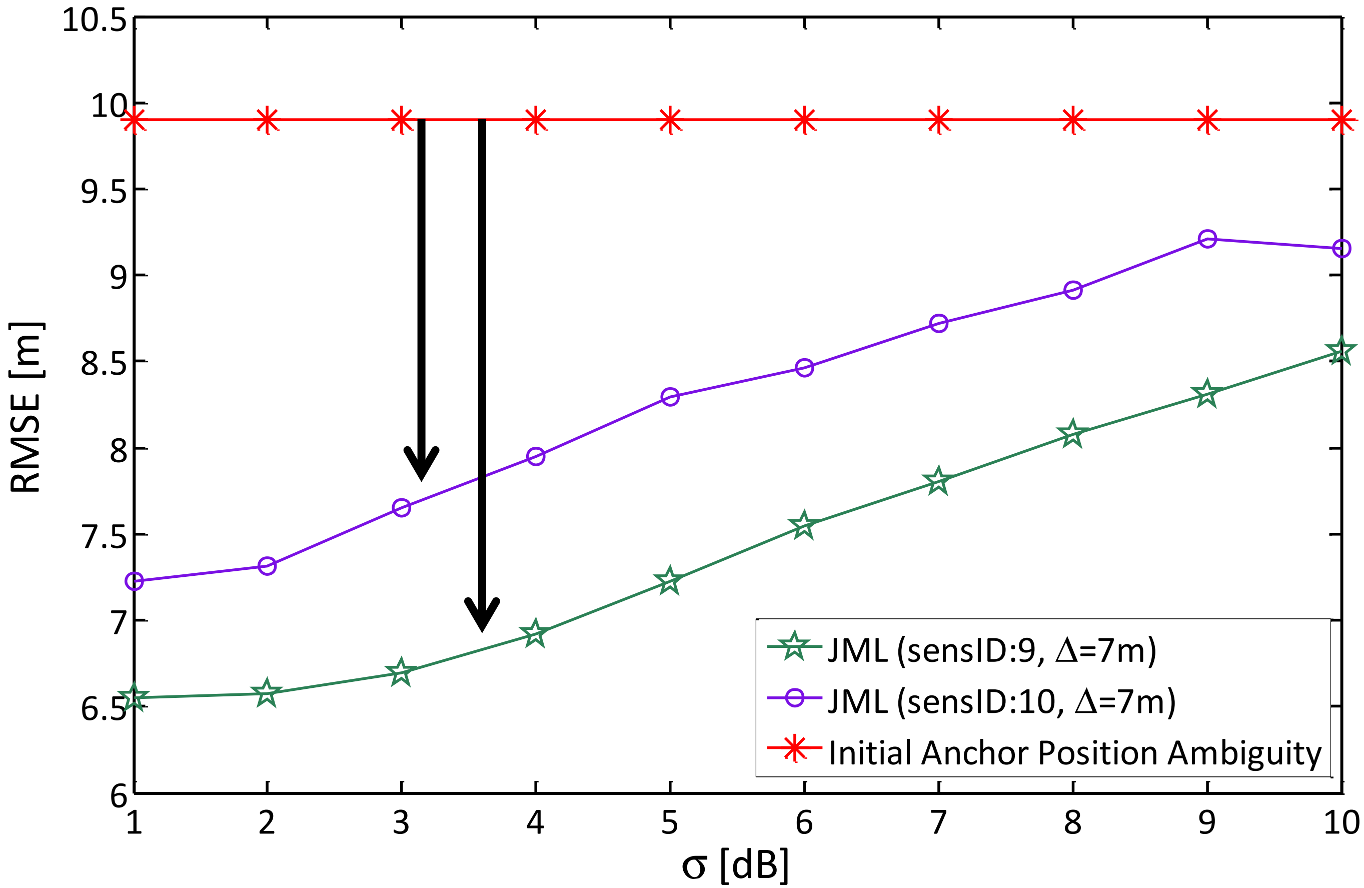
$$RMSE\left(\hat{\mathbf{r}}_i;\sigma,\Delta\right)=\sqrt{\frac{1}{L}\sum_{j=1}^{L}\left\|\hat{\mathbf{r}}_{ij}-\mathbf{r}_i\right\|^2};$$

$$\hat{\mathbf{r}}_{ij}=[\hat{\boldsymbol{\theta}}_{JMLj}]_{(2i+1):(2(i+1))};\ i=9,10$$

Initial Anchor Position Ambiguity

$$RMSE_{init}\left(\hat{\mathbf{r}}_i;\sigma,\Delta\right)=\sqrt{Tr[\mathbf{K}_i]}=\Delta\sqrt{2};\ i\in U$$

JML performance evaluation (4/4) → anchor ambiguity reduction



The JML improves the reliability of the anchor position estimates
→ ***reduced network topology ambiguity***

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Concluding remarks

- In challenging environments, the information about the sensors' positions might be unreliable → the problem can be alleviated using joint localization algorithms
- This work introduced general RSS-based localization framework using non-Bayesian estimation formalism that:
  - **Improves the transmitter localization performances**
  - **Reduces the sensors' positional ambiguity**
- The JML is introduced as a typical representative of the proposed framework
- The JML performance evaluation proves that substantial transmitter localization improvements and network ambiguity reduction gains can be attained via joint localization

THANK YOU FOR YOUR KIND ATTENTION!

QUESTIONS?

http://wingroup.feit.ukim.edu.mk

Backup slides

Theoretical performance bounds

- The **Cramer-Rao Lower Bound** (CRLB) [1] bounds the covariance matrix of any unbiased, non-Bayesian estimator of $\boldsymbol{\theta}$ via the inverse of the **Fisher Information Matrix** (FIM) $\mathbf{J}(\boldsymbol{\theta})$

$$\mathbf{C}_{\hat{\mathbf{r}}_{TX}} \geq \mathbf{J}^{-1}(\mathbf{r}_{TX}) = \boldsymbol{\Psi}^{-1}\left[\mathbf{I}_2 + \Upsilon \mathbf{J}^{-1}(\ldots,\mathbf{r}_i,\ldots)\Upsilon^T \boldsymbol{\Psi}^{-1}\right]$$

$$\mathbf{C}_{\hat{\mathbf{r}}_i} \geq \mathbf{J}^{-1}(\mathbf{r}_i) = (a_i\mathbf{R}_i + \mathbf{K}_i^{-1})^{-1}\left[\mathbf{I}_2 + a_i^2\mathbf{R}_i\mathbf{J}^{-1}(\mathbf{r}_{TX})\mathbf{R}_i(a_i\mathbf{R}_i + \mathbf{K}_i^{-1})^{-1}\right]$$

$$\boldsymbol{\Psi} = \sum_{i=1}^{n} a_i\mathbf{R}_i;\ \mathbf{R}_i = \mathbf{q}_i\mathbf{q}_i^T;\ a_i = \left(10\gamma(\sigma\Delta\ln 10 d_i)^{-1}\right)^2;\ \mathbf{q}_i = (\cos\phi_i \ \ \sin\phi_i)^T;\ \Upsilon = -(\ldots a_i\mathbf{R}_i\ldots)_{i\in U}$$

- **Objective:** **prove convergence** of the derived bounds by investigating the **asymptotic performance** of the JML
  - The JML is expected to achieve the CRLB in terms of RMSE for small environmental variability and small anchor position errors (i.e. in the **small variation/error region**)

$$RMSE(\hat{\mathbf{r}}_{TX};\sigma,\Delta) = \sqrt{\frac{1}{L}\sum_{j=1}^{L}\left\|\hat{\mathbf{r}}_{TXj} - \mathbf{r}_{TX}\right\|^2};\quad RMSE(\hat{\mathbf{r}}_i;\sigma,\Delta) = \sqrt{\frac{1}{L}\sum_{j=1}^{L}\left\|\hat{\mathbf{r}}_{ij} - \mathbf{r}_i\right\|^2}$$

$$PEB(\mathbf{r}_{TX};\sigma,\Delta) = \sqrt{Tr[\mathbf{J}^{-1}(\mathbf{r}_{TX})]};\quad PEB(\mathbf{r}_i;\sigma,\Delta) = \sqrt{Tr[\mathbf{J}^{-1}(\mathbf{r}_i)]}$$

RMSE: simulation

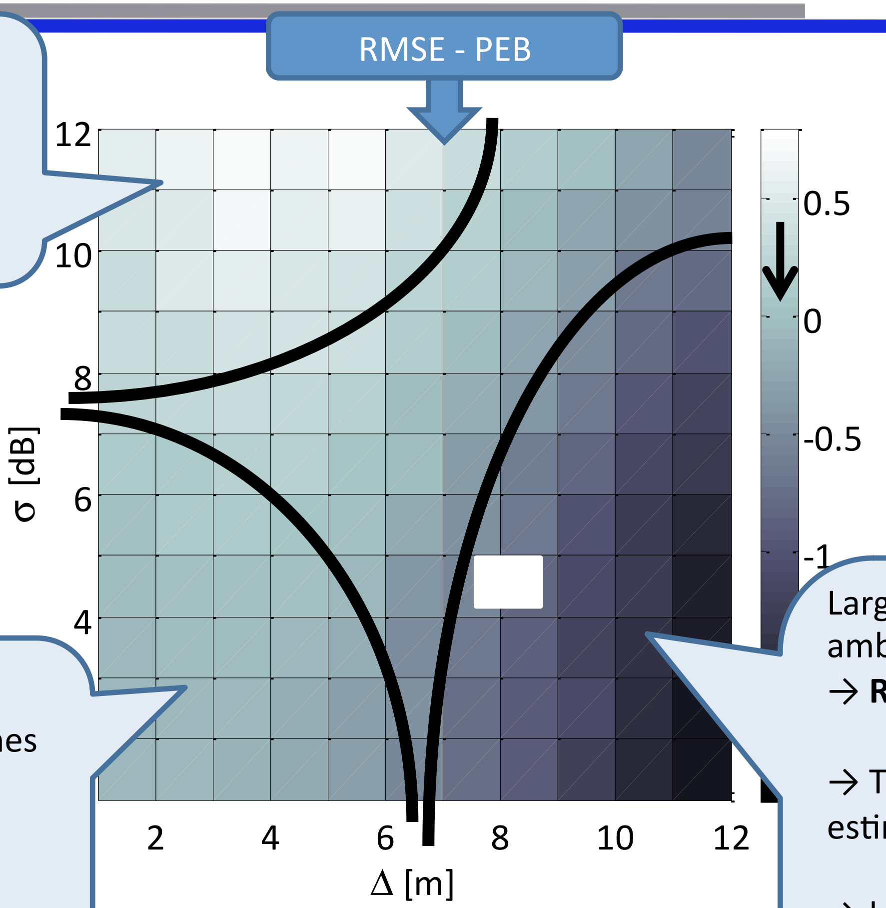
Position Error Bound (PEB): theoretical

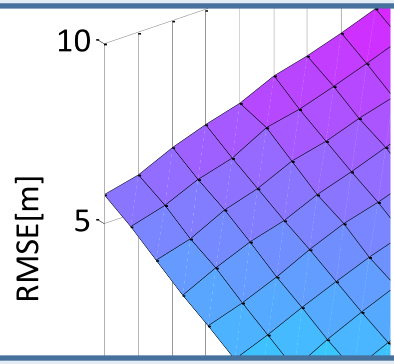
[1] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory, Prentice-Hall, 1998.

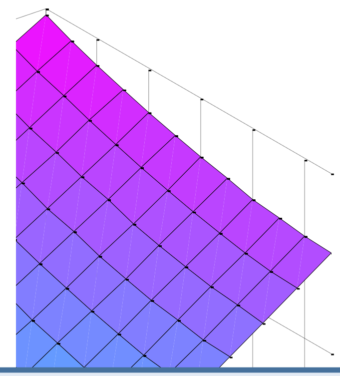
Bound convergence (1/3)
→ transmitter localization

High radio environmental variability region

→ **RMSE – PEB > 0**







Small variations region
→ **RMSE - PEB ≈ 0** (approaches zero)

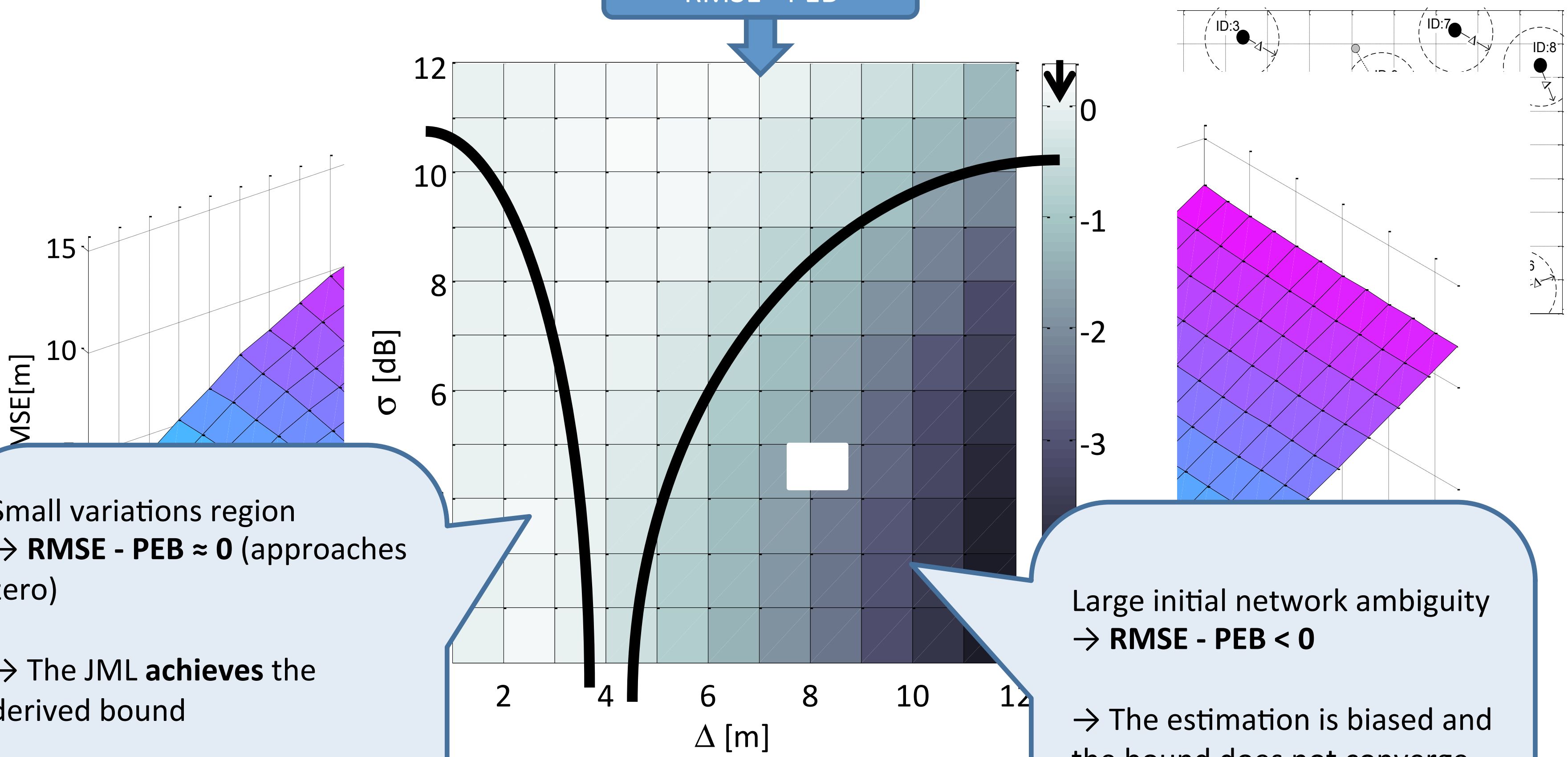
→ The JML **achieves** the derived bound

Large initial network ambiguity
→ **RMSE - PEB < 0**

→ Typical behavior for **biased** estimation

→ large initial ambiguities introduce bias in the estimation process and JML performs better than expected in terms of RMSE

Bound convergence (2/3) → anchor localization (ID:2)



Small variations region
→ **RMSE - PEB ≈ 0** (approaches zero)

→ The JML **achieves** the derived bound

→ The Bound converges even for large environmental variability

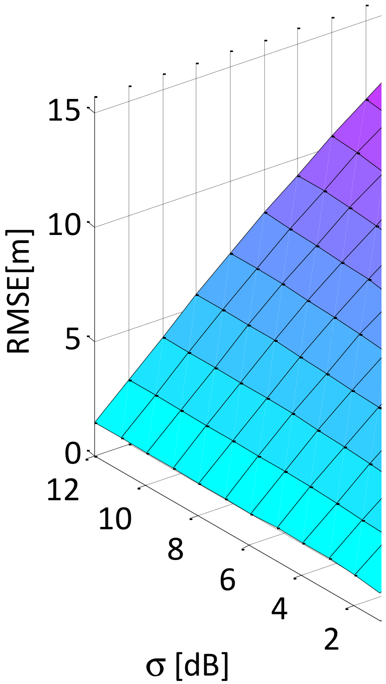
Large initial network ambiguity
→ **RMSE - PEB < 0**

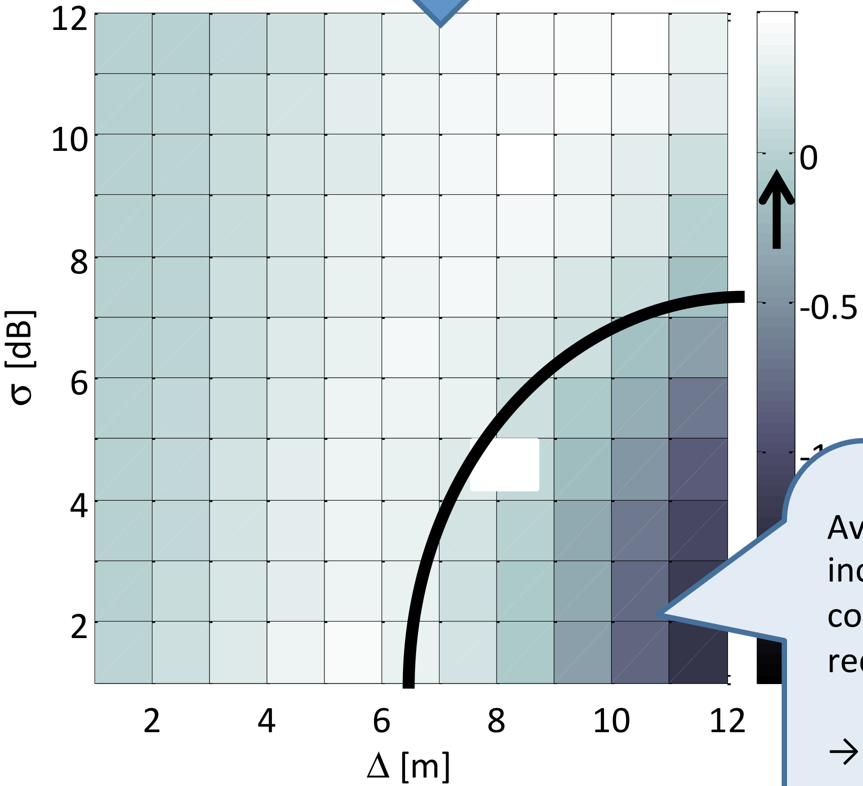
→ The estimation is biased and the bound does not converge

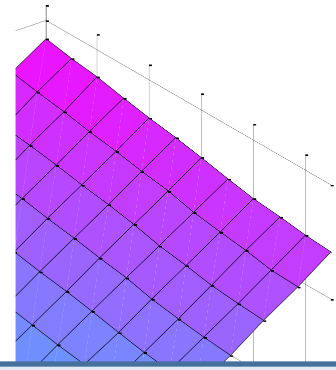
→ Notice that **ID:2** is **very close to the transmitter**

Bound convergence (3/3)
→ anchor localization (average)

RMSE - PEB







Averaging over sensors increases the region of convergence of the bound and reduces the bias.

→ The bound for far anchors converges in larger region compared with close anchors

Joint Maximum Likelihood

- The main task of the RSS-based joint localization framework is to **estimate $\boldsymbol{\theta}$ based on the information contained in $\mathbf{X}$** [1]
- Employing **Maximum Likelihood** (ML) approach [1], results in the **Joint Maximum Likelihood** (JML) localization algorithm

$$\hat{\boldsymbol{\theta}}_{JML} = \arg\max_{\boldsymbol{\theta}} \left\{ \ln p(\mathbf{X};\boldsymbol{\theta}) \right\} = \arg\min_{\boldsymbol{\theta}} \left\{ -\ln p(\mathbf{X};\boldsymbol{\theta}) \right\}$$

where $p(\mathbf{X};\boldsymbol{\theta})$ is **the joint probability density function** of the data vector $\mathbf{X}$

- Employing the path loss and the anchor position estimate distribution **assumptions**, the log-likelihood function $L(\boldsymbol{\theta}|\mathbf{X})=\ln p(\mathbf{X};\boldsymbol{\theta})$ results in

$$L(\boldsymbol{\theta} \mid \mathbf{X}) = \ln p(\mathbf{X};\boldsymbol{\theta}) = -\frac{1}{\sigma^2}\sum_{i=1}^{n}\left(P_i - \delta_i(\boldsymbol{\theta})\right)^2 - \sum_{i \in U}\left(\tilde{\mathbf{r}}_i - \mathbf{r}_i\right)^T \mathbf{K}_i^{-1}\left(\tilde{\mathbf{r}}_i - \mathbf{r}_i\right)$$

- When $\tilde{\mathbf{r}}_i = \mathbf{r}_i;\ i \in N$, then $\boldsymbol{\theta} = \mathbf{r}_{TX}$ and the JML becomes the **Legacy ML** (LML) localization algorithm

$$\hat{\boldsymbol{\theta}}_{LML} = \arg\min_{\boldsymbol{\theta}} \left\{ \sum_{i=1}^{n}\left(P_i - \delta_i(\boldsymbol{\theta})\right)^2 \right\}$$

[1] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory, Prentice-Hall, 1998.

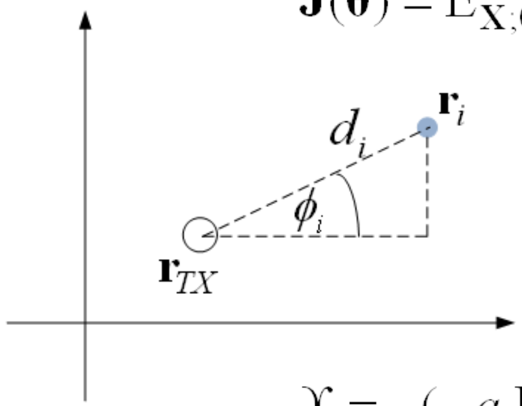
Performance bounds (1/2)

- The **Cramer-Rao Lower Bound** (CRLB) [1] bounds the performance of any unbiased, non-Bayesian estimator of $\boldsymbol{\theta}$ via the inverse of the **Fisher Information Matrix** (FIM)
- The **information inequality** provides the CRLB estimation bound for $\boldsymbol{\theta}$ [1]

$$\mathbf{C}_{\hat{\boldsymbol{\theta}}} \geq \mathbf{J}^{-1}(\boldsymbol{\theta})$$

  - The information inequality **bounds the covariance matrix** of any unbiased estimator of deterministic parameters with the inverse of the FIM calculated for the respective parameters
- Fisher Information Matrix for the parameter vector $\boldsymbol{\theta}$
  - Quantifies **the information about the unknown parameter** $\boldsymbol{\theta}$ vector contained in the **observed data** $\mathbf{X}$

$$\mathbf{J}(\boldsymbol{\theta}) \triangleq \mathrm{E}_{\mathbf{X};\boldsymbol{\theta}} \left\{ \left(\frac{\partial \ln p(\mathbf{X};\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right)^T \frac{\partial \ln p(\mathbf{X};\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right\} = \begin{pmatrix} \boldsymbol{\Psi} & \Upsilon \\ \Upsilon^T & \mathbf{D}+\boldsymbol{\Delta} \end{pmatrix}$$



$$\boldsymbol{\Psi} = \sum_{i=1}^{n} a_i \mathbf{R}_i ; \ \mathbf{R}_i = \mathbf{q}_i \mathbf{q}_i^T$$

$$a_i^2 = 10\gamma(\sigma \Delta \ln 10)^{-1}$$

$$\mathbf{q}_i = (\cos\phi_i \ \ \sin\phi_i)^T$$

$$\Upsilon = -(\ldots a_i \mathbf{R}_i \ldots)_{i\in U} ; \ \mathbf{D} = diag\{a_i \mathbf{R}_i\}_{i\in U} ; \ \boldsymbol{\Delta} = diag\{\mathrm{K}_i^{-1}\}_{i\in U}$$

[1] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory, Prentice-Hall, 1998.

Performance bounds (2/2)

- The **Equivalent FIM** (EFIM) can be calculated as Schur's complement of the FIM
  - The positional information can be studied **separately** for each node in the network

$$\mathbf{J}(\mathbf{r}_{TX}) = \boldsymbol{\Psi} - \Upsilon\left(\mathbf{D}+\boldsymbol{\Delta}\right)^{-1}\Upsilon^{T} = \sum_{i=1}^{n} a_i \mathbf{R}_i - \sum_{i\in U} a_i^2 \mathbf{R}_i \mathbf{K}_i \mathbf{R}_i + \sum_{i\in U} \frac{a_i^3 \mathbf{R}_i \mathbf{K}_i \mathbf{R}_i \mathbf{K}_i \mathbf{R}_i}{1 + a_i \mathbf{q}_i^T \mathbf{K}_i \mathbf{q}_i}$$

$$\mathbf{J}(\mathbf{r}_i) = \left(a_i \mathbf{R}_i + \mathbf{K}_i^{-1}\right) - a_i^2 \mathbf{R}_i \boldsymbol{\Psi}^{-1} \mathbf{R}_i - a_i^2 \mathbf{R}_i \boldsymbol{\Psi}^{-1} \Upsilon_{U/\{i\}} \mathbf{J}_{U/\{i\}}^{-1}(\ldots,\mathbf{r}_i,\ldots) \Upsilon_{U/\{i\}}^{T} \boldsymbol{\Psi}^{-1} \mathbf{R}_i ;\; i \in U$$

- The CRLB for each network node (transmitter and sensors) is obtained by employing the EFIM expressions in the information inequality

$$\mathbf{C}_{\hat{\mathbf{r}}_{TX}} \geq \mathbf{J}^{-1}\left(\mathbf{r}_{TX}\right) = \boldsymbol{\Psi}^{-1}\left[\mathbf{I}_2 + \Upsilon \mathbf{J}^{-1}(\ldots,\mathbf{r}_i,\ldots)\Upsilon^{T}\boldsymbol{\Psi}^{-1}\right]$$

$$\mathbf{C}_{\hat{\mathbf{r}}_i} \geq \mathbf{J}^{-1}\left(\mathbf{r}_i\right) = \left(a_i \mathbf{R}_i + \mathbf{K}_i^{-1}\right)^{-1}\left[\mathbf{I}_2 + a_i^2 \mathbf{R}_i \mathbf{J}^{-1}\left(\mathbf{r}_{TX}\right) \mathbf{R}_i \left(a_i \mathbf{R}_i + \mathbf{K}_i^{-1}\right)^{-1}\right]$$