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Sensing Period Optimization in
Primary-Secondary IEEE 802.22 Networks

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INTRODUCTION

Dynamic Spectrum Access (DSA) constitutes a set of approaches to spectrum reform. The main motivations behind DSA are inefficient regulatory regime currently in use and the physical limits on the spectrum useful for mobile terrestrial communications. Despite other models available, the hierarchical access model has been receiving particular attention because it allows to Primary Users and Secondary Users: the former correspond to incumbent, legacy services that have regulatory right to use one or more spectrum pools, while the latter are envisioned as cognitive device aimed at exploiting idle spectrum opportunities without harming incumbent communications. Under this model, one can rely on spectrum underlay, i.e., Ultra Wide Band Communications, or Opportunistic Spectrum Access (OSA). OSA is the approach of choice in this project because it represents an important application scenario for cognitive radio, which empowers secondary users with spectrum sensing capabilities. In particular, to render feasible the exploitation of TV white spaces the IEEE 802.22 working group is working towards the standardization of Wireless Regional Area Networks. License holders to TV bands have priority access to spectrum and are referred to as Primary Users. Wireless Regional Area Networks are allowed to operate in overlay with Primary Users provided that they make use of TV white spaces on a non-interfering basis.

This work of thesis is concerned with the settings that a Wireless Regional Area Networks needs perform to comply with the 802.22’s functional requirements. Emphasis is put on the mandatory application of Dynamic Frequency Selection improved with Cognitive Radio and Cooperative Spectrum Sensing.

A cognitive radio can understand the context it finds itself in and autonomously configure itself in response to a set of goals. Though other methods are available, cognitive radios usually perform sensing tasks to determine whether spectrum is occupied. Nevertheless, an individual cognitive radio has to cope with fading, shadowing, and penetration losses. These local detection issues can be mitigated by using cooperative spectrum sensing, in which the Wireless Regional Area Networks
base station and its served customer premise equipment share sensing information to get a more accurate picture of current spectrum occupancy.

The design of sensing mechanisms for Wireless Regional Area Networks has aroused considerable research interest. Most approaches proposed have been based on the enforcement of quiet periods, during which all transmissions cease and focus is placed on spectrum sensing. The current 802.22 standard draft adopts a two-stage sensing mechanism, whose key aspect is the trade-off between sensing accuracy and data rate. In the first stage, short quiet periods can be used with minimal degradation of the Wireless Regional Area Networks data rate. The second stage involves one long quiet period, scheduled only if a larger number of samples is needed to improve sensing accuracy. The Two Stage Sensing mechanism being considered for standardization is an enhanced version of the C-MAC protocol. However, the emphasis has been put on Single-Stage Sensing with arbitrary allocation and duration of quiet periods. It has been suggested that it is possible to determine the optimum duration of quiet periods, i.e., the optimum sensing time, in the sense that the average data rate can be maximized. This issue has been formulated as an optimization problem and solved using numerical optimization. Though the optimization of the duration of quiet periods has been treated in a rather complete fashion, the literature for Wireless Regional Area Networks still have not managed to bring together all relevant criteria, e.g., the 802.22 MAC structure, DFS timing requirements, characteristics of the TV channel, mandatory sensitivity according to primary user type, benefits and drawbacks from Cooperative Spectrum Sensing.

In generally, there are three major spectrum sensing methods, including matched filter, energy detection and feature detection. In this work, we consider the most common and simplest method: Energy Detector.

The rest of the thesis is organized as follows. The Cognitive Radio Technology, the Dynamic Spectrum Access and Spectrum Sensing Methods are described in Chapters 1, 2 and 3, respectively. In Chapter 4 system parameters of energy detector and their relationship to each other are analyzed. To do this we consider a less complex spectrum sensing methods, in which do not to rely on quiet periods. Then, the optimization of sensing period in Local and Cooperative Spectrum Sensing is investigate in Chapters 5 and 6, respectively.
Chapter 1

COGNITIVE RADIO TECHNOLOGY

The proliferation of wireless communication services caused a concurrent increase in the demand for and congestion of radio frequency (RF) spectrum. This congestion put a premium on the cost of spectrum and has created a battle between the public, private, and military sectors over frequency ownership. Studies have shown, however, that spectral utilization is relatively low when examined not just by frequency domain, but across the spatial and temporal domains.

In November 2002, the Federal Communications Commission (FCC) published a report prepared by the Spectrum-Policy Task Force, aimed at improving the way in which this precious resource is managed in United States. The spectrum allocation chart used by the Federal Communications Commission is show in figure 1 and it seems to indicate a high degree of utilization.

![Figure 1: FCC spectrum allocation chart](image-url)
However, the FCC Spectrum Policy Task Force reported vast temporal and geographic variation in the usage of the allocated spectrum with use ranging from 15% to 85% in the bands below 3 GHz that are favoured in non-line-of-sight radio propagation, as show in the following figure.

![Figure 2: Spectrum utilization](image)

Indeed, scanning portions of the radio spectrum, we would find that [1,2]:

- some frequency bands in the spectrum are largely unoccupied most of the time;
- some other frequency bands are only partially occupied;
- the remaining frequency bands are heavily used

The underutilization of the electromagnetic spectrum leads to think in terms of Spectrum Holes [1]:

“A spectrum holes is a band of frequencies assigned to a primary user, at a particular time and specific geographic location, the band is not being utilized by that user”.
Although the static spectrum assignment policy generally worked well in the past, there have been a dramatic increase in the access to limited spectrum to mobile services and applications in recent years. The limited available spectrum due to the nature of radio propagation and the need for more efficiency in the spectrum usage necessitates a new communication paradigm to exploit the existing spectrum opportunistically. Inspired by the successful global use of multi-radio co-existing at 2.4 GHz unlicensed ISM bands and other, dynamic spectrum access is proposed as a solution to problems of current inefficient spectrum usage. The inefficient usage of the existing spectrum can be improved through opportunistic access to the licensed bands by existing users.

The key enabling technology of dynamic spectrum access is *Cognitive Radio* (CR) technology, which provides the capacity to share the wireless channel with the licensed users in an opportunistic way.
1.1 From Software Defined Radio to Cognitive Radio

Many different wireless communication systems exist and they are widely used for different purpose and application scenarios. As show in Figure 1, there are some popular wireless communications international standards, ranging from body area networks, personal area networks, local area networks and metropolitan area networks, to wide area networks, with different applications scenarios and optimised system parameters. As a matter of fact, for the cellular type system alone, there are a variety of system in use, such as legacy GSM, GPRS and EDGE, 3GPP wideband code division multiple access (WCDMA) with its update versions of HSDPA and HSUPA, and the upcoming 3GPP long-term evolution (LTE), and that is just considering air-interface technology.

All these air-interface technology may co-exist in different geographical regions, and may co-exist simultaneously at the same location. So, a flexible realisation of terminal devices to allow users to appropriately use wireless communications is definitely essential.
Since the early days of electronic communication, however, typical (wireless) communication system have been implemented by certain dedicated hardware and likely dedicated application specific integrated chips (ASIC) based on specific system parameters designed for use. Progress in digital processing technology has led to a new concept, known as Software Defined Radio (SDR), in which the radio functions are defined by software.

A rigorous definition of the concept of Software Defined Radio does not yet exist. Some definitions often found in the literature [3]:

- Flexible TX / RX architecture, controlled and programmable by software;
- Signal processing able to replace the radio functionality;
- Software realization of terminals “multiple mode/standard”;
- "Air interface downloadability": radio equipment dynamically reconfigurable by downloadable software at every level of the protocol stack;
- Transceiver where the following can be defined by software:
  - Frequency band and radio channel bandwidth;
  - Modulation and coding scheme;
  - Radio resource and mobility management protocols;
  - User applications.

These parameters can be adapted and changed by the network operator, the service provider or the final user.

In summary, the following definition could be used:

"Software Defined Radio is an emerging technology, thought to build flexible radio systems, multi service, multiband, reconfigurable and reprogrammable by software".

Another definition was proposed by Wireless Innovation Forum, working in collaboration with the Institute of Electrical and Electronic Engineers (IEEE) P1900.1 group [4]:

“Software Defined Radio is a Radio in which some or all physical layer functions are software defined”.

This opens the possibility of defining, in software, the typical functions of a radio interface which is usually implemented by dedicated hardware in the transmit and receive equipment. In other words, SDR is a collection of hardware and software technologies that enable reconfigurable system architectures. Indeed, some or all of the radio’s operating functions (also referred to as physical layer processing) are implemented through modifiable software or firmware operating on programmable processing technologies. These devices include field programmable gate arrays (FPGA), digital signal processors (DSP), general purpose processors (GPP), programmable System on Chip (SoC) or other application specific programmable processors. The use of these technologies allows new wireless features and capabilities to be added to existing radio systems without requiring new hardware.

SDR provides an efficient and comparatively inexpensive solution to the challenge of building multi-mode, multi-band, multi-functional wireless devices that can be adapted, updated, or enhanced by using software upgrades.

The SDR concept can also be used to implement mobile terminals and base stations. In the case of mobile terminals, the SDR concept enables them to adapt dynamically to the local radio environment. In the case of base stations, the SDR concept is aimed at configuring a common platform for a specific air interface by downloading the appropriate software. This can be done either during operation or as part of the production cycle.

Users will benefit from the SDR system by having access to different networks and various air interfaces using a single terminal. Access will be possible to global networks, corporate networks and domestic networks. The user will also benefit from the downloading of services, features and applications. As a result of the terminal’s flexibility, it provides continuity of service when the transmission standards change in the coverage area.

SDR puts the service provider in the position of being able to offer attractive new services and features “on the fly”. The consequences will be a significant improvement in service quality, increased traffic and more revenue.
The software radio concept should also be regarded as a way of making users, service providers and manufacturers more independent of standards. The benefits of this approach are that air interfaces may, in principle, be tailored to the needs of a particular service for a particular user in a given environment at a given time.

The concept of Cognitive Radio (CR) emerged as an extension of SDR technology. In particular, a Cognitive Radio is an SDR that additionally senses its environment, tracks changes, and reacts upon its findings.

Experts agree that a CR device should have the following characteristics:

- aware of its environment;
- capable of altering its physical behaviour to adapt to its current environment;
- learns from previous experiences;
- deals with situations unknown at the time of the radio’s design.

A precious definition of Cognitive radio does not yet exist. Indeed, while many researchers and public officials agree that upgrading a software radio’s control processes will add significant value to software radio, there is currently some
disagreement over how much “cognition” is needed which results in disagreement over the precise definition of a cognitive radio.

The term “Cognitive Radio” was coined by Joseph Mitola III in an article published in 1999 [5]. He defines the Cognitive Radio as “A radio that employs model based reasoning to achieve a specified level of competence in radio-related domains”.

Then, in his publication [6] Haykin defines a cognitive radio as “An intelligent wireless communication system that is aware of its surrounding environment (i.e., outside world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real time, with two primary objectives in mind: highly reliable communications whenever and wherever needed; efficient utilization of the radio spectrum”.

Another definitions was proposed by IEEE USA, FCC and IEEE 1900.1 group. The IEEE USA offered the following definition [7]: “A radio frequency transmitter/receiver that is designed to intelligently detect whether a particular segment of the radio spectrum is currently in use, and to jump into (and out of, as necessary) the temporarily-unused spectrum very rapidly, without interfering with the transmissions of other authorized users”.

After two years, FCC defined the Cognitive Radio as [8] ”a radio that can change its transmitter parameters based on interaction with the environment in which it operates”.

The IEEE tasked the IEEE 1900.1 group to define cognitive radio which has the following working definition: “A type of radio that can sense and autonomously reason about its environment and adapt accordingly. This radio could employ knowledge representation, automated reasoning and machine learning mechanisms in establishing, conducting, or terminating communication or networking functions
with other radios. Cognitive radios can be trained to dynamically and autonomously adjust its operating parameters.”

These is some of many definitions proposed for cognitive radio, but all of these assume that cognition will be implemented as a control process, presumably as part of a Software Defined Radio. Second, all of the definitions at least imply some capability of autonomous operation. Finally, the following are some general capabilities found in all of the definitions:

- Observation: whether directly or indirectly, the radio is capable of acquiring information about its operating environment;
- Adaptability: the radio is capable of changing its waveform\(^1\);
- Intelligence: the radio is capable of applying information towards a purposeful goal.

So, a mobile terminal with cognitive radio capabilities can sense the communication environments (e.g. spectrum holes, geographic location, available wire/wireless communication system or networks, available services), analyze and learn information from the environments with user’s preferences and demands, and reconfigure itself by adjusting system parameters conforming to certain policies and regulations.

\(^1\)Waveform: a protocol that specifies the shape of an electromagnetic signal intended for transmission by radio.
1.2 Cognition Cycle

As shown in the previous section, there are some differences in the definitions for CR, and they can be largely attributed to differences in the expectations of the functionality that a CR will exhibit. In his dissertation [10], J. Mitola III considers the nine levels of increasing cognitive radio functionality, ranging from a software radio to a complex self-aware radio, as shown in Table 1.

<table>
<thead>
<tr>
<th>Level</th>
<th>Capability</th>
<th>Task Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Pre-programmed</td>
<td>The radio has no model-based reasoning capability (SDR)</td>
</tr>
<tr>
<td>1</td>
<td>Goal-driven</td>
<td>Goal-driven choice of RF band, air interface, and protocol</td>
</tr>
<tr>
<td>2</td>
<td>Context Awareness</td>
<td>Infers external communications context (minimum user involvement)</td>
</tr>
<tr>
<td>3</td>
<td>Radio Aware</td>
<td>Flexible reasoning about internal and network architectures</td>
</tr>
<tr>
<td>4</td>
<td>Capable of Planning</td>
<td>Reasons over goals as a function of time, space, and context</td>
</tr>
<tr>
<td>5</td>
<td>Conducts Negotiations</td>
<td>Expresses arguments for plans/alternatives to user, peers, networks</td>
</tr>
<tr>
<td>6</td>
<td>Learns Fluents</td>
<td>Autonomously determines the structure of the environment</td>
</tr>
<tr>
<td>7</td>
<td>Adapts Plans</td>
<td>Autonomously modifies plans as learned fluents change</td>
</tr>
<tr>
<td>8</td>
<td>Adapts Protocols</td>
<td>Autonomously proposes and negotiates new protocols</td>
</tr>
</tbody>
</table>

Table 1: Level of Cognitive Radio functionality [10]

Furthermore, if the radio is to be aware, it must interact with the outside world. As a reference for how a Cognitive Radio could achieve these levels of functionality and how it may interact with the environment is accomplished via the cognition cycle (Figure 5).
In the cognition cycle, a radio receives information about its operating environment (Outside world) through direct observation or through signaling. This information is then evaluated (Orient) to determine its importance. Based on this valuation, the radio determines its alternatives (Plan) and chooses an alternative (Decide) in a way that presumably would improve the valuation. Assuming a waveform change was deemed necessary, the radio then implements the alternative (Act) by adjusting its resources and performing the appropriate signaling. These changes are then reflected in the interference profile presented by the cognitive radio in the Outside world. As part of this process, the radio uses these observations and decisions to improve the operation of the radio (Learn), perhaps by creating new modeling states, generating new alternatives, or creating new valuations.

In other words, a Cognitive Radio behaves according to five main actions:

- OBSERVE: CRs are aware of their surrounding environment;
- PLAN: CRs evaluate among several strategies
- DECIDE: CRs are always capable to select one strategy of operation;
- LEARN: CRs can enrich experience by forming new strategies;
- ACT: CRs perform communication according to the selected strategy.

As the learning process can be quite cycle intensive and is not necessary for many of the envisioned applications and as artificial intelligence is not yet ripe for deployment, many researchers have assumed lower levels of functionality in their cognitive radio. For instance, in his remarks at the 2005 MPRG Technical Symposium, Bruce Fette, Chief Scientist at General Dynamics Decision Systems, noted that many members of the defence community refer to the cognition cycle as the “OODA” loop – emphasizing only the observation, orientation, decision, and action portions cognition cycles. Even the source of the most expansive interpretation of cognitive radio [10] suggests that learning would occur during sleep or “prayer” (insight gained from external entities) epochs and that during wake epochs the cognitive radio would primarily operate as an OODA loop augmented by some light planning capabilities.
1.3 The Capability of Cognitive Radios

The capabilities of Cognitive Radios as node of Cognitive Radio Network can be classified according to their functionalities based on the definition of cognitive radio. A CR shall the environment (cognitive capability), analyse and learn sensed information (self-organised capability) and adapt to environment (reconfigurable capabilities).

1.3.1 Cognitive Capability

Cognitive capability refers to the ability of the radio technology to capture or sense the information from its radio environment. This capability cannot simply be realized by monitoring the power in some frequency band of interest but more sophisticated techniques are required in order to capture the temporal and spatial variations in the radio environment and avoid interference to other users. Through this capability, the portions of the spectrum that are unused at a specific time or location can be identified. Consequently, the best spectrum and appropriate operating parameters can be selected.

The task required for adaptive operation in open spectrum [6] are shown in Figure 7, which is referred to as Cognitive Cycle.
• **Spectrum sensing**: a CR monitors the available spectrum bands, and then detect spectrum holes. It could incorporate a mechanism that would enable sharing of the spectrum under the terms of an agreement between a licensee and third party.

• **Spectrum analysis**: The characteristics of the spectrum holes that are detected through spectrum sensing are estimated.

• **Spectrum decision**: a CR determines the data rate, the transmission mode, and the bandwidth of the transmission. Then, the appropriate spectrum band is chosen according to the spectrum characteristics and user requirements.

• **Location identification**: location identification is the ability to determine its location and the location of other transmitter, and then select the appropriate operating parameters such as the power and frequency allowed at its location. In bands as those used for satellite (receive-only), location technology may be an appropriate method of avoiding interference because sensing technology would not be able to identify the locations of nearby receivers.

• **Network/system discovery**: For a cognitive radio terminal to determine the best way to communicate, it shall first discover available networks around it. These networks are reachable either via directed one hop communication or via multi-hop relay nodes. The ability to discovery one hop or multi-hop away access networks is important.
- **Service discovery**: service discovery usually accompanies with network/system discovery. Network or system operators provide their services through their access networks. A cognitive radio terminal shall find appropriate services to fulfill its demands.

### 1.3.2 Reconfigurable Capability

Reconfigurability is the capability of adjusting operating parameters for the transmission on the fly without any modifications on the hardware components. This capability enables the cognitive radio to adapt easily to the dynamic radio environment.

- **Frequency agility**: It is the ability of a radio to change its operating frequency. This ability usually combines with a method to dynamically select the appropriate operating frequency based on the sensing of signals from other transmitters or on some other method.

- **Dynamic Frequency Selection**: this is defined in the rules as a mechanism that dynamically detects signals from other radio frequency systems and avoids co-channel operation with those systems. The methods that a device could use to decide when to change frequency or polarization could include spectrum sensing, geographic location monitoring, or an instruction from a network or another device.

- **Adaptive Modulation/Coding**: it can modify transmission characteristics and waveforms to provide opportunities for improved spectrum access and more intensive use of spectrum while “working around” other signals that are present. A cognitive radio could select the appropriate modulation type for use with a particular transmission system to permit interoperability between systems.
Transmit Power Control: this is a feature that enables a device to dynamically switch between several transmission power levels in the data transmission process. It allows transmission at the allowable limits when necessary, but reduces the transmitter power to a lower level to allow greater sharing of spectrum when higher power operation is not necessary.

Dynamic System/Network Access: for a cognitive radio terminal to access multiple communication systems/networks which run different protocols, the ability to reconfigure itself to be compatible with these systems is necessary.

The transmission parameters of a cognitive radio can be reconfigured not only at the beginning of a transmission but also during the transmission. According to the spectrum characteristics, these parameters can be reconfigured such that the cognitive radio is switched to a different spectrum band, the transmitter and receiver parameters are reconfigured and the appropriate communication protocol parameters and modulation schemes are used.

1.3.3 Self-organised Capability

Cognitive radios should be able to self-organise their communication based on sensing and reconfigurable functions.

Spectrum/Radio Resource Management: to efficiently manage and organize spectrum holes information among cognitive radios, good spectrum management scheme is necessary.

Mobility and Connection Management: due to the heterogeneity of CRNs, routing and topology information is more and more complex. Good mobility and connection management can help neighborhood discovery, detect available Internet access and support vertical handoffs, which help cognitive radios to select route and networks.
Trust/Security Management: since CRNs are heterogeneous networks in nature, various heterogeneities (e.g. wireless access technologies, system/network operators) introduce lots of security issues. Trust is thus a prerequisite for securing operations in CRNs.
REFERENCES


Chapter 2

DYNAMIC SPECTRUM ACCESS

The underutilization of spectrum as revealed by extensive measurements of actual spectrum usage has stimulated exciting activities in the engineering, economics, and regulation communities in searching for better spectrum management policies and techniques. Hence, in the opposite to the current static spectrum management policy, the term Dynamic Spectrum Access was coined.

As show in the following figure, dynamic spectrum access strategies can be broadly categorized under three models.

![Figure 8: Dynamic Spectrum Access](image)

*Figure 8: Dynamic Spectrum Access*
2.1 Dynamic Exclusive Use Model

This model maintains the basic structure of the current spectrum regulation policy: spectrum bands are licensed to services for exclusive use. The main idea is to introduce flexibility to improve spectrum efficiency. Two approaches have been proposed under this model [3]:

1. *Spectrum Property Rights*: it allows licensees to sell and trade spectrum and to freely choose technology. Economy and market will thus play a more important role in driving toward the most profitable use of this limited resource. Note that even though licensees have the right to lease or share the spectrum for profit, such sharing is not mandated by the regulation policy [1] [2].

2. *Dynamic Spectrum Allocation*: it aims to improve spectrum efficiency through dynamic spectrum assignment by exploiting the spatial and temporal traffic statistics of different services. In other words, in a given region and at a given time, spectrum is allocated to services for exclusive use. This allocation, however, varies at a much faster scale than the current policy [3] [4].

Based on an exclusive-use model, these approaches cannot eliminate white space in the spectrum resulting from the bursty nature of wireless traffic.
2.2 Open Sharing Model

This model is also referred as *Spectrum Commons* and it employs open sharing among peer users as the basis for managing a spectral region. Advocates of this model draw support from the phenomenal success of wireless services operating in the unlicensed industrial, scientific, and medical (ISM) radio band (e.g., WiFi). Centralized, and distributed spectrum sharing strategies have been initially investigated to address technological challenges under this spectrum management model.

2.3 Hierarchical Access Model

Built upon a hierarchical access structure with primary and secondary users, this model can be considered as a hybrid model of the above two. The basic idea is to open licensed spectrum to secondary users and limit the interference perceived by primary users.

Two approaches to spectrum sharing between primary and secondary users have been considered: *Spectrum Underlay* and *Spectrum Overlay*.

2.3.1 Spectrum Underlay

In an Underlay System, regulated spectral mask impose stringent limits on radiated power of secondary users as a function of frequency. Hence, secondary users must operate below the noise floor of primary users.

Radios coexist in the same band with primary licensees, but are regulated to cause interference below prescribed limits. For example, a low-powered radio could coexist in the same frequency channel with a high-powered broadcast radio. An example is the UWB communication that uses a spreading transmitted signals over a
wide frequency band. Hence, the secondary users can potentially achieve short-range high data rate with extremely low power transmission power, as show in the following figure.

![Figure 9: Example of Spectrum Underlay](image)

The problem is the following: if primary users transmit all the time, this approach doesn’t rely on detection and exploitation of spectrum white space.

### 2.3.2 Spectrum Overlay

Spectrum overlay was first envisioned by Mitola under the term “Spectrum Pooling” and later investigated by the DARPA XG program under the term “Opportunistic Spectrum Access (OSA)”.

In opposite to Spectrum Underlay, this approach doesn’t necessarily impose severe conditions on the power transmission of secondary users, but rather on where on when they may transmit. The scope of this approach is to target the spatial and temporal white space by allowing to permit to secondary users to identify and exploit local and instantaneous spectrum availability in a non intrusive manner.
Compared to the Dynamic Exclusive Use and Open Sharing models, this hierarchical model is perhaps the most compatible with the current spectrum management policies and legacy wireless systems. Furthermore, the underlay and overlay approaches can be employed simultaneously to further improve spectrum efficiency. Hence, we focus on this model, in particular on the Opportunistic Spectrum Access.
2.4 Opportunistic Spectrum Access

Spectrum Overlay, or Opportunistic Spectrum Access (OSA), can be applied in either temporal or spatial domain. In the former, secondary users aim to exploit temporal spectrum opportunities resulting from the bursty traffic of primary users. In the latter, secondary users aim to exploit frequency bands that are not used by primary users in a particular geographic area. A typical application is the reuse of certain TV-bands that are not used for TV broadcast in a particular region. In the TV broadcast system, TV-bands assigned to adjacent regions are different to avoid co-site interference.

Basic components of OSA include:

- Spectrum Opportunity Identification: this module is responsible for accurately identifying and intelligently tracking idle frequency bands that are dynamic in both time and space. It is crucial to OSA in order to achieve non intrusive communication
- Spectrum Opportunity Exploitation: this module takes input from the opportunity identification module and decides whether and how a transmission should take place.
- Regulatory Policy: it defines the basic etiquette for secondary users to ensure compatibility with legacy systems.

To illustrate the basic technical issue in OSA, we can consider the following example of OSA network. We can consider a spectrum consisting of N channels, and these are allocated to a network of primary users. Channel can be a frequency band with certain bandwidth, a collection of spreading codes in a CDMA network, or a set of tones in an OFDM system. Furthermore, we assume that cross-channel interference is negligible. Thus, a secondary user transmitting over an available channel does not interfere with primary users using other channels.

The N channels are allocated to a network of primary users and we assume that the primary system uses a synchronous slot structure, although the basic ideas apply
more generally. The traffic statistic of the primary system are such that the occupancy of these N channels follows a Markov process with $2^N$ states, where the state is defined as the availability (idle or busy) of each channel. Hence, with this model of primary network, a secondary users seek spectrum opportunities in these N channels independently. In each slot, a secondary user choose a channel to sense and decides whether to access based on imperfect sensing outcomes.

In the following sections we analyse the basic components of OSA, but before a definition of Spectrum Opportunity is necessary.

### 2.4.1 Definition of Opportunity

Intuitively, a channel can be considered as an opportunity if it is not currently used by primary users. As show in the following figure, we have different resources that can be considered as opportunity, i.e., some resources are frequency, time, geographical space, code.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>What needs to be sensed?</th>
<th>Comments</th>
<th>Illustrations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Opportunity in the frequency domain.</td>
<td>Availability in part of the frequency spectrum. The available spectrum is divided into narrower chunks of bands. Spectrum opportunity in this dimension means that all the bands are not used simultaneously at the same time, i.e., some bands might be available for opportunistic usage.</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>Opportunity of a specific band in time.</td>
<td>This involves the availability of a specific part of the spectrum in time. In other words, the band is not continuously used. There will be times where it will be available for opportunistic usage.</td>
<td></td>
</tr>
<tr>
<td>Geographical space</td>
<td>Location (latitude, longitude, and elevation) and distance of primary users.</td>
<td>The spectrum can be available in some parts of the geographical area while it is occupied in some other parts at a given time. This takes advantage of the propagation loss (path loss) in space. These measurements can be avoided by simply looking at the interference level. No interference means no primary user transmission in a local area. However, one needs to be careful because of hidden terminal problem.</td>
<td></td>
</tr>
<tr>
<td>Code</td>
<td>The spreading code, time hopping (TH), or frequency hopping (FH) sequences used by the primary users. Also, the timing information is needed so that secondary users can synchronize their transmissions w.r.t. primary users. The synchronization estimation can be avoided with long and random code usage. However, partial interference in this case is unavoidable.</td>
<td>The spectrum over a wideband might be used at a given time through spread spectrum or frequency hopping. This does not mean that there is no availability over this band. Simultaneous transmission without interfering with primary users would be possible in code domain with an orthogonal code with respect to codes that primary users are using. This requires the opportunity in code domain, i.e., not only detecting the usage of the spectrum, but also determining the used codes, and possibly multipath parameters as well.</td>
<td></td>
</tr>
</tbody>
</table>

Figure 11: example of Spectrum Opportunities
For example, with the help of next figure where A is the transmitter and B its intended receiver, we identify conditions for a channel to be considered as an opportunity.

![Illustration of spectrum opportunity](image)

**Figure 12: Illustration of spectrum opportunity**

In this case, a channel is an opportunity to A and B if they can communicate successfully over this channel while limiting the interference to primary users below a prescribe level. This means that A will not interfere with primary receivers and B will not be influenced by primary transmitters. Hence, if we consider a monotonic and uniform signal attenuation, and omni-directional antennas, a channel is an opportunity to A and B if no primary users are receiving within a distance $r_{tx}$ from A and no primary users are transmitting within a distance $r_{rx}$ from B. The distance $r_{tx}$ is determined by the transmission power of secondary user and the maximum allowable interference to primary users, while $r_{rx}$ is determined by the transmission power of primary users and the secondary user’s interference tolerance. So, the Spectrum Opportunity is a local concept defined with respect to a particular pair of secondary users, and it is determined by their position and by the communication activities of primary users.
REFERENCES

As has been pointed out in the previous sections, a Cognitive Radio has to be able to sense the environment over a wide portion of the spectrum and autonomously adapt to it since the Cognitive Radio does not have rights to any frequency bands. This task performed by Cognitive Radio is known as Spectrum Sensing [1, 2, 3] (or Spectrum Monitoring [4, 5]). Generally speaking, Spectrum Sensing in wireless communications is one of the most challenging tasks that a Cognitive Radio has to perform. Depending on the required level of automation and self-management capabilities, Spectrum Sensing has to provide to the Cognitive Radio different information in order to predict the radio spectrum utilization. For these reasons, in some applications, providing information only about the frequency usage would not be sufficient, and other characteristics about the portion of the spectrum under investigation have to be provided in order to predict the radio spectrum utilization (e.g. number of transmitted signals, carrier frequency, power, transmission technique, modulation, etc). In fact, prior knowledge about the transmitted signal and its parameters (e.g. carrier frequency, power, modulation, etc.) is usually not available. Moreover, received signals are corrupted by channel distortions (e.g. severe multipath fading), and spread spectrum transmission techniques are often used in order to obtain a low probability of interception. Hence, the goal of spectrum sensing is to decide between the following hypothesis:

1. $H_0$: Primary user is absent;
2. $H_1$: Primary user is present.

in order to avoid the harmful interference to the primary system. This behavior is referred to as detecting free bands, which meaning is to identify frequency bands...
which are free of already established communications. Free band detection can be illustrated as in the next figure.

![Free Band detector architecture](image.png)

Radio signal $y(t)$ received at the antenna is first filtered on a bandwidth $BL$, then down converted to baseband digitized before being sent to the detector. Finally, a decision is made on whether the band $BL$ should be considered as « free » or « occupied », based on this computation.

In the following subsections a survey on these Spectrum Sensing techniques will be provided and advantages/disadvantage for the different approaches will be discussed. Before to start,

### 3.1 Matched Filter

Using a matched filter is the optimal solution to signal detection in presence of noise [6] as it maximizes the received Signal-to-Noise Ratio ($SNR$). It is a coherent detection method, which necessitates the demodulation of the signal, which means that cognitive radio equipment has the a priori knowledge on the received signal(s), e.g. order and modulation type, pulse shaping filter, data packet format, etc. Most often, telecommunication signals have well-defined characteristics, e.g. presence of a pilot, preamble, synchronization words, etc., that permit the use of these detection techniques. Based on a coherent approach, matched filter has the advantage to only require a reduces set of samples, function of $O(1/SNR)$, in order to reach a
convenient detection probability [6]. If $X[n]$ is completely known to the receiver then the optimal detector for this case is:

$$T[y] = \sum_{n=0}^{N-1} y[n]x[n]^{H0}$$

(1)

If $\gamma$ is the detection threshold, then the number of samples required for optimal detection are

$$N = \left[Q^{-1}(P_d) - Q^{-1}(P_f)\right]^2 SNR^{-1} = O(SNR^{-1})$$

(2)

where $P_d$ and $P_f$ are the probabilities of detection and false alarm respectively. Hence, the main advantage of matched filter is that thanks to coherency it requires less time to achieve high processing gain since only $O(SNR^{-1})$ samples are needed to meet a given probability of detection constraint.

However, a significant drawback of a matched filter is that a cognitive radio would need a dedicated receiver for every signal it may have to detect. Thus in the case of multi-waveform detection, this approach is often not used.

**3.2 Energy Detector**

One approach to simplify matched filtering is to perform non-coherent detection through energy detection [6]. This sub-optimal technique has been extensively used in radiometry. Energy detection is a well known detection method mainly because of its simplicity. The basic functional method involves a squaring device, an integrator and comparator.
It can be implemented either in time domain or in frequency domain. Time domain implementation would require front-end filtering of the signal to be detected (primary signal) before the squaring operation.

In frequency domain implementation, after front-end band-pass filtering, the received signal samples are converted to frequency domain samples using Fourier transform. Signal detection is then effected by comparing the energy of the signal samples falling within certain frequency band with that of a threshold value.

The threshold value is an ambient noise power arising from the receiver itself and RF interference in the surrounding.

Energy detection or radiometer method lies on a stationary and deterministic model of the signal mixed with a stationary white Gaussian noise with a known single-side power spectrum density $\sigma_0$. A simplified diagram of a radio meter is shown in the next figure.

![Figure 14: Energy detector scheme](image)

To detect a weak primary signal confined inside some a priori known bandwidth $B$, one could pose as a binary hypothesis testing problem as follows:

$$
    H_0 : x(n) = v(n) \\
    H_1 : x(n) = s(n) + v(n),
$$

where $H_0$ represents the absence of the primary signal, i.e., the received baseband complex signal $x(n)$ contains only additive white Gaussian noise (AWGN), and $H_1$ represents the presence of the primary signal, i.e., $x(n)$ consists of a primary signal $s(n)$ corrupted by $v(n)$. Moreover, $N$ corresponds to the number of available measurements. The signal is detect by comparing the output of the energy detector with a threshold which depends on the noise floor. Energy detection method is a non-
coherent energy detector and one of simplest approach for deciding between two hypothesis: $H_0$ and $H_1$.

Set $x = [x(1), x(2), \ldots, x(N)]^T$ and $s = [s(1), s(2), \ldots, s(N)]^T$ respectively the received signal and the primary signal vectors, the decision rules is given by

$$T(x) = \sum_{n=1}^{N} |x(n)|^2 > \gamma$$

where $T(x)$ is the test statistic and $\gamma$ is the corresponding test threshold. Although $T(x)$ has a chi-square distribution, according to the central limit theorem $T(x)$ is asymptotically normally distributed if $N$ is a large enough. Specially, for large $N$, we can model the statistics of $T(x)$ as follows:

$$T(x) \approx \begin{cases} N(N\sigma_s^2, 2N\sigma_s^4) & \text{under } H_0 \\ N(\frac{N\sigma_s^2 + Np_s, 2N\sigma_s^4 + 4N\sigma_s^2 p_s}{N}) & \text{under } H_1 \end{cases}$$

where $p_s = \|s\|/N$ represents the average primary signal power. In this way, for large $N$, the Probability of False Alarm and the Probability of Detection, can be approximated, respectively, as

$$P_f = P(H_0 \mid H_1) = Q\left(\frac{\gamma - N\sigma_s^2}{\sigma_s^2 \sqrt{2N}}\right)$$

$$P_d = P(H_1 \mid H_1) = Q\left(\frac{\gamma - N\sigma_s^2 - Np_s}{\sigma_s \sqrt{2N\sigma_s^2 + 4Np_s}}\right)$$

where $Q(x)$ is the tail of a zero-mean unit variance Gaussian random variable, and it’s given by
\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{\tau^2}{2}} d\tau \] (7)

Moreover, indicating with

\[ SNR = \frac{P_x}{\sigma_v^2} = \frac{\|S\|^2}{N\sigma_v^2} \] (7)

the Signal-to-Noise Ratio, using relations of Probability of False Alarm and Probability of Detection, it is easy to see that in order to ensure a particular operation point \((P_d, P_f)\), the required number of samples, \(N\), is given by

\[ N = 2\left[Q^{-1}(P_f) - Q^{-1}(P_d)\sqrt{1 + 2SNR}\right]^2 SNR^{-2} \] (8)

Hence, in the large Signal-to Noise Ratio regime (i.e. \(SNR >> 1\)) we conclude that \(O(1/SNR)\) samples are needed to meet the desired operation point \((P_d, P_f)\), and \(O(1/SNR^2)\) are needed in the low regime (i.e. \(SNR << 1\)).

There are several drawbacks of energy detectors that might diminish their simplicity in implementation. First, a threshold used for primary user detection is highly susceptible to unknown or changing noise levels. Even if the threshold would be set adaptively, presence of any in-band interference would confuse the energy detector. Furthermore, in frequency selective fading it is not clear how to set the threshold with respect to channel notches. Second, energy detector does not differentiate between modulated signals, noise and interference. Since, it cannot recognize the interference, it cannot benefit from adaptive signal processing for cancelling the interferer. Furthermore, spectrum policy for using the band is constrained only to primary users, so a cognitive user should treat noise and other secondary users differently. Lastly, an energy detector does not work for spread spectrum signals: direct sequence and frequency hopping signals, for which more sophisticated signal processing algorithms need to be devised. In general, we could
increase detector robustness by looking into a primary signal footprint such as modulation type, data rate, or other signal feature.

### 3.3 Cyclostationary Feature Detection

An alternative method for the detection of primary signals is *Cyclostationary Feature Detection* [6]. Modulated signals are in general coupled with sine wave carriers, pulse trains, repeated spreading, hopping sequences, or cyclic prefixes which result in built-in periodicity. These modulated signals are characterized as cyclostationary because their mean and autocorrelation exhibit periodicity. This periodicity is introduced in the signal format at the receiver so as to exploit it for parameter estimation such as carrier phase, timing or direction of arrival. These features are detected by analyzing a spectral correlation function (SCF). The main advantage of this function is that it differentiates the noise from the modulated signal energy. This is due to the fact that noise is a wide-sense stationary signal with no correlation however modulated signals are cyclostationary due to embedded redundancy of signal periodicity. Analogous to autocorrelation function, spectral correlation function can be defined as

\[
S_X(f) = \lim_{T \to \infty} \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} X_T(t, f + \frac{\alpha}{2})^* X_T^*(t, f - \frac{\alpha}{2}) dt, \tag{9}
\]

where the finite time Fourier Transform is given by

\[
X_T(t, f) = \int_{t-T/2}^{t+T/2} x(u) e^{-j2\pi fu} du \tag{10}
\]

A simplified diagram of a cyclostationary feature detected is shown in the next figure
Unlike power spectrum density, which is real-valued one dimensional transform, the spectral correlation function is two dimensional transform, in general complex-valued and the parameter $\alpha$ is called cycle frequency. Power spectral density (PSD) is a special case of a spectral correlation function for $\alpha=0$.

Because of the inherent spectral redundancy signal selectivity becomes possible. Analysis of signal in this domain retains its phase and frequency information related to timing parameters of modulated signals. Due to this, overlapping feature in power spectral density are non overlapping feature in cyclic spectrum. Hence different types of modulated signals that have identical power spectral density can have different cyclic spectrum.

Because of all these properties cyclostationary feature detector can perform better than energy detector in discriminating against noise. However it is computationally complex and requires significantly large observation time.
REFERENCES


Chapter 4

PERFORMANCE ANALYSIS OF ENERGY DETECTOR: EPISODIC ENVIRONMENT

This chapter is concerned with less complex spectrum sensing methods, which do not to rely on quiet periods. Therefore, secondary communications are not interrupted while sensing is performed. Since the choice of one action in each observation time does not depend on the previous one, the task environment 1 (ENV1) is assumed episodic. Under this assumptions we analyse the performance of the following sensing architectures:

1. Local Spectrum Sensing (LSS);
2. Distributed Spectrum Sensing (DSS).

As we have seen in the previous chapter, the goal of Spectrum Sensing is to detect a weak primary signal confined inside some a priori known bandwidth $B$, one could pose as a binary hypothesis testing problem as follows:

$$
\begin{align*}
H_0 : x(n) = v(n) \\
H_1 : x(n) = s(n) + v(n), \quad n = 1,2,\ldots,N
\end{align*}
$$

(1)

where $H_0$ represents the absence of the primary signal, i.e., the received baseband complex signal $x(n)$ contains only additive white Gaussian noise (AWGN), and $H_1$ represents the presence of the primary signal, i.e., $x(n)$ consists of a primary signal $s(n)$ corrupted by $v(n)$. In this work, we use the radiometer technique known as Energy Detection. It is the most common way of spectrum sensing because of its low computational and implementation complexities. In addition, it is more generic as receivers do not need any knowledge on the primary users’ signal. As seen, the Probabilities of False Alarm and Detection can be defined as follow:
\[ P_f = P(H_0 \mid H_1) = Q \left( \frac{\gamma - N \sigma_v^2}{\sigma_v^2 \sqrt{2N}} \right) \]  \hspace{1cm} (2)

\[ P_d = P(H_1 \mid H_0) = Q \left( \frac{\gamma - N \sigma_v^2 - N p_s}{\sigma_v^2 \sqrt{2N \sigma_v^2 + 4N p_s}} \right) \]  \hspace{1cm} (3)

respectively. It is easy to see that in order to ensure a particular operation point \((P_{d}, P_{f})\), the required number of samples, \(N\), is given by

\[ N = 2 \left[ Q^{-1}(P_f) - Q^{-1}(P_d) \sqrt{1 + 2SNR} \right]^2 SNR^{-2} \]  \hspace{1cm} (4)

As shown in the next figure, in the large Signal-to-Noise Ratio regime (i.e. \(SNR \gg 1\)) we conclude that \(O(1/SNR)\) samples are needed to meet the desired operation point \((P_f, P_d)\), and \(O(1/SNR^2)\) are needed in the low regime (i.e. \(SNR \ll 1\)).

![Figure 16: Number of samples versus Signal-to-Noise Ratio (operation point: \(P_f=0.1, P_d=0.9\))](image-url)
4.1 Local Spectrum Sensing

In this subsection we analyze the performance of Local Spectrum Sensing (LSS). In other words, we consider a single CR and it takes a decision independently by the other CRs, only with its environments observations.

Performance of a detection algorithm can be summarized with two probabilities: Probability of Detection and Probability of False Alarm. Using relation of number of samples, it’s easy to see that the two previous probabilities are given by

\[ P_d = Q\left(\frac{Q^{-1}(P_f) - \sqrt{\frac{N \cdot SNR^2}{2}}}{\sqrt{1 + 2SNR}}\right) \]  
\[ P_f = Q\left(\frac{Q^{-1}(P_d) \sqrt{1 + 2SNR} + \sqrt{\frac{N \cdot SNR^2}{2}}}{\sqrt{1 + 2SNR}}\right) \]

Moreover, there is a third probability that we use in the next of this work, known as Probability of MisDetection. In particular this is the probability that sensing algorithm decides for spectrum busy when this is idle, and it’s given by

\[ P_m = 1 - P_d = 1 - Q\left(\frac{Q^{-1}(P_f) - \sqrt{\frac{N \cdot SNR^2}{2}}}{\sqrt{1 + 2SNR}}\right) \]

As show in the previous section, the number of samples required for a certain operation point \((P_f, P_d)\) depends on SNR value, but how many time is necessary to collect these samples? Hence, first of all we analyze the number of samples as a function of SNR and we compute the Sensing Time as a function of the Number of Sample. Sensing Time is given by
\[ T = \frac{N}{B} \] (8)

where \( N \) is the number of samples and \( B \) is the bandwidth. In this work we assume that \( B = 6\text{MHz} \). Then we plot the Receiver Operating Characteristic (ROC) curves (Probability of Detection versus Probability of False Alarm) and the Complementary ROC curves (Probability of MisDetection versus Probability of False Alarm).

### 4.1.1 Probability of Detection Versus Number of Samples

In this section we analyze the number of sample required to guarantee a certain operation point \((P_f, P_d)\). We consider a range of SNR from -20db to 20dB. In order to obtain a fit analysis, we divide the operation Signal-to Noise Ratio in three ranges:

1. Low Range: \(-20 \leq \text{SNR} \leq -10\);
2. Medium Range: \(-5 \leq \text{SNR} \leq 5\);
3. High Range: \(10 \leq \text{SNR} \leq 20\).

Moreover, value of the Probability of False Alarm is fixed to \(P_f=0.1\). As show previously, the equation of the number of sample is given by (4) and is valid for a Number of Samples larger than 20. For this reason, we must consider the following graphics only for a large \( N \), e.g., \( N > 20 \).

**LOW RANGE**

We can see that the number of samples required increase with the decrease of Signal to Noise Ratio. In fact, for an operation point \((P_f=0.9, P_d=0.1)\) and SNR=-20dB we need \(1.3270e+005\) samples and so 11.1msec to collected them.
MEDIUM RANGE

As expected, when SNR increase the number of sample required decrease. In particular, with a with a value of SNR = 5 dB the number of samples necessary is $\theta(1/\text{SNR})$. Using equation (8), for a SNR = 0 dB and for an operation point ($P_f=0.1$, $P_d=0.9$), Sensing Time is $T = 0.0020$ ms.
HIGH RANGE

In this case, we can show that the number of samples is under the threshold’s value (N>20). With a number of samples N>20 we achieve a high Probability of Detection, i.e., \( P_d = 1 \).

![Figure 19: Number of Samples versus Probability of Detection – High SNR(\( P_f = 0.1 \))](image)

4.1.2 Probability of MisDetection Versus Number of Samples

As for the Probability of Detection, we can compute the number of samples required to guarantee a certain Probability of MisDetection. In this case, we consider a fixed Probability of False Alarm (\( P_f = 0.1 \)) and compute the Probability of MisDetection using the equation (7). Moreover, in order to obtain a fit analysis we use the same range defined above.
LOW RANGE

Figure 20: Probability of MisDetection versus Number of Samples – Low SNR (Pf=0.1)

MEDIUM RANGE

Figure 21: Probability of MisDetection versus Number of Samples – Medium SNR (Pf=0.1)
4.1.3 Sensing Time

In this subsection we analyze the sensing time required to collect the number of samples necessary for a certain operation point according with equation (8). In this analysis we consider a fixed Probability of False Alarm ($P_f=0.1$) and a variable Probability of Detection. In this way, we can see how the SNR values influence on the Sensing Time. Since we know that an higher number of samples are required for low SNR, we consider this value from -23dB to -10dB. The result is shown in the following figure.
We can see that below a SNR=−20dB the detection becomes progressively harder and at -23dB signal cannot be detect regardless of the sensing time duration. Logically, the Time Sensing required depends by the operation point. As shown in the next figure, we can define a \( SNR_{wall} \), below of this the detection become “impossible” in terms of Sensing Time.
4.1.4 Receiver Operation Characteristic

In this section we want analyse the probability of Detection as function of Probability of False Alarm by varying SNR, it is called the Receiver Operation Characteristics (ROC). Using this curves, it’s possible to determine the performance of a particular application. In other words, fixed the values of SNR and \( P_f \) we can determine the respective value of Probability of Detection.

Since we have seen that Probability of Detection depends on Number of Samples, we consider the Medium Range of SNR: \((-5 \leq \text{SNR} \leq 5\): using equation (8) and an operation point \((P_f = 0.1, P_d = 0.9)\) we obtain N=24 (SensingTime = 0.0204 ms).

The result is shown in the following figure.

![ROC – Probability of Detection versus Probability of False Alarm](image)

---

If the Probability of False Alarm and Probability of Detection increase simultaneously and if the SNR is greater that -5dB, regardless of the Probability of False Alarm the energy detector is optimum.
4.1.5 Complementary Receiver Operation Characteristic

In this section we want analyse the Probability of MisDetection as function of Probability of False Alarm, hence the Complementary Receiver Operation Characteristic. As done for ROC curves we consider the Medium Range: $(-5 \leq SNR \leq 5)$ and so a number of Samples N=24. Using equation (7) we obtain the following curves.

![Figure 24: ComplementaryROC: Probability of MisDetection versus Probability of False Alarm](image)

As expected, the Probability of MisDetection assumes very small values when the Probability of False Alarm is near to 1. Indeed, since Probability of False Alarm is very close to 1, the Probability of Detection is nearly 1, as shown by ROC curves.
4.1.6 Behaviour in SNR

The goal of this section is to analyze the behaviour of Energy Detection method in function of Signal-to-Noise Ratio. In other words we want understand how the value of SNR influences the performance of detection. The following analysis is divided in two parts, each of which involve both the Probability of Detection and the Probability of MisDetection. The first covers the analysis of these probability for a fixed Probability of False Alarm and for a variable Number of Samples. Instead, in the second the Number of Samples is set a variable Probability of False Alarm is considered. Furthermore, since the Number of Samples depends on the value of SNR, we consider separately the three ranges defined above.

4.1.6.1 Probability of Detection versus Signal to Noise Ratio

I CASE

In this case we consider a fixed Probability of False Alarm and a variable Number of Samples. In particular we consider

- Probability of False Alarm: \( P_f = 0.1 \)
- Number of Samples: \( N = \{5, 15, 50, 100, 500, 1000\} \).
As expected, the Probability of Detection increase with the SNR. Moreover, we can see that, for a fixed value of SNR, this probability increase with the number of samples.

II CASE

In this second case we want analyse the Probability of Detection as function of SNR for a variable Probability of False Alarm. Since we have seen that Probability of Detection depends by Number of Samples, we consider three different ranges and three different values of Number Samples. As in the “Number of Samples versus Probability of Detection” section, the three ranges are:

1. Low Range \((-20 \leq SNR \leq -10\)): N=3000 (SensingTime = 1.1293 ms);
2. Medium Range: \((-5 \leq SNR \leq 5\)): N=24 (SensingTime = 0.0204 ms);
3. High Range: \((10 \leq SNR \leq 20\)): N=2 (SensingTime = 0.02 µs).

All values are computed for an operation point \((P_f=0.1, P_d=0.9)\) and the medium values of SNR of each range.
Figure 26: Probability of Detection versus SNR (Low SNR range, fixed $N = 3000$)

Figure 27: Probability of Detection versus SNR (Medium SNR range, fixed $N = 24$)
Previous figures show that the Probability of Detection increase when Probability of False Alarm also increase. Moreover, the performance of energy detector is excellent when the SNR is greater than -5 dB.

4.1.6.2 Probability of MisDetection versus Signal to Noise Ratio

I CASE
In this case we consider a fixed Probability of False Alarm and a variable Number of Samples. In particular we consider

- Fixed Probability of False Alarm: \( P_f = 0.1 \)
- Variable Number of Samples: \( N = [5, 15, 50, 100, 500, 1000] \).
II CASE

In this second case we want analyse the probability of MisDetection as function of SNR for a variable Probability of False Alarm. Since we have seen that Probability of MisDetection depends by Number of Samples, we consider three different range an so three different values of samples. As in the “Number of Samples vs Probability of Detection” section, the three range are:

4. Low Range ($-20 \leq \text{SNR} \leq -10$): $N=3000$ (SensingTime = 1.1293 ms);
5. Medium Range: ($-5 \leq \text{SNR} \leq 5$): $N=24$ (SensingTime = 0.0204 ms);
6. High Range: ($10 \leq \text{SNR} \leq 20$): $N=2$ (SensingTime = 0.02 µs).

All values are computed for an operation point ($P_f=0.1$, $P_d=0.9$) and the medium values of SNR of each range.
Figure 30: Probability of MisDetection versus SNR (Low SNR range, fixed $N = 3000$)

Figure 31: Probability of MisDetection versus SNR (Medium SNR range, fixed $N = 24$)
Figure 32: Probability of MisDetection versus SNR (High SNR range, fixed $N = 2$)
4.2 Cooperative Spectrum Sensing

In order to improve the performance of the spectrum sensing, the Secondary Users can cooperate to detect the presence of the Primary User. The decision topology used for cooperative detection is a parallel network with a fusion center as shown in the next figure.

This topology consists of $N \geq 2$ local detectors all observing the same phenomenon. The local detectors transmit their measurement statistics to a fusion center which makes a global decision.

In this section we consider **Hard Decision Fusion** which means that each secondary user makes a local decision about the presence of primary user and then sends the binary decision to the fusion center for decision fusion. We consider performance’s analysis in the case of secondary users grouped in clusters, hence all with the same average Signal-to-Noise Ratio.

Before to introduce the performance measurements of the Hard Decision Fusion, we start to analyse the collaborative spectrum sensing problem over a single narrow band. Consider a Cognitive Radio Network with $M$ secondary users. Each user $i$, with $i = 1,2,\ldots,M$, collects $N$ measurements and formulates the binary hypothesis test problem:
\[ H_0 : x_i(n) = v_i(n) \]
\[ H_1 : x_i(n) = h_i s(n) + v_i(n), \quad n = 1, 2, \ldots, N \] (9)

where \( h_i(n) \) is the channel gain between the primary user and the \( i \) th secondary user. Without loss of generality, it is assumed that \( h_i \) is constant during the detection interval (\( N \) samples) and the value of \( N \) should be much less than the coherence time of the channel between the primary user and the secondary receivers.

With energy detection, secondary user \( i \) uses the following decision rule:

\[
T_i(x_i) = \sum_{n=1}^{N} |x_i(n)|^2 \begin{array}{ll} \geq & H_1 \\ < & H_0 \end{array} \quad i = 1, 2, \ldots, M (10)
\]

where \( x_i = [x_i(1), x_i(2), \ldots, x_i(N)]^T \), \( T_i(x_i) \) measure the total energy and \( \gamma_i \) is the local threshold at the \( i \) th secondary user.

To evaluate the sensing performance, we define the probability of correctly detecting spectral holes and the probability of interference as:

\[
P(H_0 \mid H_0) = 1 - P_f \] (11)

\[
P(H_0 \mid H_i) = 1 - P_d \] (12)

where \( P_f \) and \( P_d \) denote the probability of false alarm and the probability of detection respectively. Specifically, \( P(H_0 \mid H_0) \) is the probability that the secondary users successfully identify the unoccupied spectral segment, while \( P(H_0 \mid H_i) \) measures the probability that a secondary users cause harmful interference to the primary user (Probability of MisDetection).
4.2.1 Hard Decision Fusion

**Hard Decision Fusion** means that each user observes, in the case of energy detection, the signal energy in a given spectrum band, compares it to a threshold and makes a decision on the presence of a primary user according to the observation. Each cooperative node then shares its decision with other radios using zero or one to inform whether they observe a free channel or an occupied channel, respectively.

In the fusion center there is a fusion of decision according to a specific criterion. It is known that approaches based on likelihood-ratio test (LRT) provide the optimal performance according to the Neyman-Pearson criterion. Denote the decision from the individual nodes by a binary vector $u = [u_1, u_2, \ldots, u_M]^T$, where

$$
    u_i = \begin{cases} 
    0 & \text{if the } i\text{th node decides } H_0 \\
    1 & \text{if the } i\text{th node decides } H_1 
    \end{cases}
$$

(13)

Let $P(u|H_0)$ and $P(u|H_1)$, respectively, represent the probability distribution functions of $u$ under the hypothesis $H_0$ and $H_1$. Then the LRT detector is given by

$$
    L(u) = \frac{P(u | H_1)}{P(u | H_0)} \begin{cases} 
    > \gamma^* & \text{if } H_1 \\
    < \gamma^* & \text{if } H_0 
    \end{cases}
$$

(14)

where $\gamma^*$ is the optimal threshold determined by the target probability of detecting the spectral hole. Computing the optimal local decision thresholds $\{\gamma_i\}$ under the Neyman-Pearson criterion is mathematically untractable, and the problem becomes NP-complete if the measurements at the individual nodes are correlated. Hence, in this section we consider suboptimal solutions.

The suboptimal decision from cooperating radios can be combined in several different ways. Let $i$, with $i = 1, 2, \ldots, M$, the number of cooperating users, we have:
**OR – rule:** the decision is made that a primary user is present if *one of the cooperating radios detects a primary user.* In case all secondary users have the same individual $P_d$ and $P_f$, the joint probabilities of detection $Q_{d,OR}$ and false alarm $Q_{f,OR}$ can therefore be given as

$$Q_{d,OR} = 1 - (1 - P_d)^M$$

$$Q_{f,OR} = 1 - (1 - P_f)^M$$

If each secondary user has different individual $P_d$ and $P_f$, the previous joint probabilities are given by

$$Q_{d,OR} = 1 - \prod_{i=1}^{M} (1 - P_{d,i})$$

$$Q_{f,OR} = 1 - \prod_{i=1}^{M} (1 - P_{f,i})$$

where $P_{f,i}$ and $P_{d,i}$ are the probabilities of false alarm and detection of $i$th user, respectively.

**AND – rule:** the decision that a primary user is present is made *only if all cooperating users detect the presence of primary user.* In case all secondary users have the same individual $P_d$ and $P_f$, joint probabilities of detection $Q_{d,AND}$ and false alarm $Q_{f,AND}$ for $M$ cooperating users using AND-rule can be calculated as

$$Q_{d,AND} = P_d^M$$

$$Q_{f,AND} = P_f^M$$
If each secondary user has different individual $P_d$ and $P_f$, the previous joint probabilities are given by

$$Q_{d,\text{AND}} = \prod_{i=1}^{M} P_{d,i}$$  \hspace{1cm} (21)$$

$$Q_{f,\text{AND}} = \prod_{i=1}^{M} P_{f,i}$$  \hspace{1cm} (22)$$

where $P_{f,i}$ and $P_{d,i}$ are the probabilities of false alarm and detection of $i$th user, respectively.

**MAJORITY – rule**: the decision that a primary user is present is made *if at least half of cooperating radios observe the presence of primary user*. In case all secondary users have the same individual $P_d$ and $P_f$, the joint probabilities of detection $Q_{d,\text{MAJ}}$ and false alarm $Q_{f,\text{MAJ}}$ for $M$ cooperating users using MAJORITY-rule can be calculated as

$$Q_{d,\text{MAJ}} = \sum_{i=0}^{\left\lceil \frac{M}{2} \right\rfloor} \left( \frac{M}{\left\lceil \frac{M}{2} \right\rfloor} \right)^{i} (1 - P_d)^{M - \left\lceil \frac{M}{2} \right\rfloor - i} P_d^{\left\lceil \frac{M}{2} \right\rfloor - i}$$  \hspace{1cm} (23)$$

$$Q_{f,\text{MAJ}} = \sum_{i=0}^{\left\lceil \frac{M}{2} \right\rfloor} \left( \frac{M}{\left\lceil \frac{M}{2} \right\rfloor} \right)^{i} (1 - P_f)^{M - \left\lceil \frac{M}{2} \right\rfloor - i} P_f^{\left\lceil \frac{M}{2} \right\rfloor - i}$$  \hspace{1cm} (24)$$

Logically, this rule is a special case of a more generalised voting rule and it has a decision threshold equal to $M/2$, where $M$ is the number of cooperating users. Different thresholds give rise to different rules, each having different performance.
4.2.2 Performance Analysis: Average SNR

In this section we analyze the performance of distributed spectrum sensing in the simple case where secondary users are clustered. Under this hypothesis the Signal-to-Noise ratio is the same for each cognitive radio. Hence, each SU $i$ has the same $P_f$ and $P_d$ of the other SUs.

As shown in the previous section, according to Hard decision fusion rules, the joint probability of detection, $Q_{d,MAJ}$, and the joint probability false alarm, $Q_{f,MAJ}$, depend on the probabilities $P_d$ and $P_f$ of single user. These individual probabilities can be calculated using the same approximation for number of samples shown in Step 1.

Hence, if we denote with $\overline{SNR}$ the average value of Signal-to-Noise Ratio we have

$$N = 2\sqrt{2erfc^{-1}(2P_f)} - \sqrt{2erfc^{-1}(2P_d)}\sqrt{1 + 2\overline{SNR}} \frac{\overline{SNR}}{2}$$

(25)

where we use complementary error function avoid confusion between the $Q$-function and the joint probabilities $Q_f$, $Q_d$ and $Q_m$. The $Q$-function is given by

$$Q(x) = \frac{1}{2}erfc\left(\frac{x}{\sqrt{2}}\right)$$

(26)

$$Q^{-1}(y) = \sqrt{2erfc^{-1}(2y)}$$

(27)

Using equation (25) the individual probabilities of detection and false alarm are given by

$$P_d = \frac{1}{2}erfc\left(\frac{\sqrt{2erfc^{-1}(2P_f)} - \sqrt{N\overline{SNR}^2}}{\sqrt{2(1 + 2\overline{SNR})}}\right)$$

(28)
4.2.2.1 Receiver Operation Characteristic

In this section we analyse the probability of Detection as function of Probability of False Alarm for different numbers of Cooperative Users. In other words, we describe how the use of Cooperation Strategies improves the performance of Spectrum Sensing.

As shown in the previous section, the decision from cooperating radios can be combined in several ways. For this reason, initially we consider each rule separately and then a comparison between the three rules.

**OR - rule**

In the OR fusion rule, to achieve a target joint probability of false alarm $Q_f,OR$ for the network, from (16) the individual secondary users’ targeted probability of false alarm $P_f$ is given by

$$P_f = \frac{1}{2} \text{erfc} \left( \sqrt{2} \text{erfc}^{-1}(2P_d) \sqrt{1 + \frac{2\text{SNR}}{N}} + \sqrt{\frac{N \text{SNR}^2}{2}} \frac{1}{\sqrt{2}} \right)$$

(29)

where $M$ is the number of cooperating users. Using $P_f$ of single user, the probability of detection $P_d$ of the cooperating users is given by, substituting (30) into (28),
\[ P_d = \frac{1}{2} \text{erfc} \left( \sqrt{2 \text{erfc}^{-1} \left( 2 \left( 1 - \frac{N \text{SNR}^2}{2(1 + 2 \text{SNR})} \right)^{\frac{1}{2}} \right)} \right) \]  

Substituting this equation into (15) we obtain the joint Probability of Detection. For fixed number of samples \( N=24 \) and an average Signal-to-Noise Ratio \( \text{SNR} = -5 dB \) we obtain the following ROC curves.

![Figure 34: ROC– joint Probability of Detection versus joint Probability of False Alarm (OR fusion rule)](image)

In this figure we can see that it’s possible to improve the performance with an increased number of cooperating users. In this way, for a fixed probability of false alarm \( Q_{f,OR} \), it’s possible increase the probability of detection \( Q_{d,OR} \) with an increased cooperating users’ number. For example, from 10 cooperating users we have almost 90% gain in accuracy over a single user, as shown in the next figure.
Moreover, the following figure shows that for each value of joint probability of false alarm the joint probability of detection is higher if the number of cooperating users is increased.

In order to do a detailed analysis, the next figure shows the behaviour of the joint probability of detection and of the probability of detection for each user as functions of number of cooperating user. This curves depend on the average SNR and on the
individual probability of false alarm. In particular, we considered $\overline{SNR} = -5 dB$ and $P_f = 0.1$.

![Joint Probability of Detection vs Number of Cooperating Users](image1)

![Probability of Detection of each user vs Number of Cooperating Users](image2)

**Figure 37: Probabilities of Detection (Qd and Pd) vs Number of Cooperating Users (OR rule)**

The previous figure shows that the benefits of cooperation can be achieved with a relative small number of cooperating users. In fact, with only 15 users we obtain a joint probability of detection equal to 0.9. Moreover, for $M > 15$ improving accuracy requires much more secondary users. Furthermore, another advantage from cooperation is that local demands imposed on individual secondary users can be relaxed for a fixed target joint $Q_d$. Indeed, for a joint probability of detection equal to 0.9, the individual probability of detection is lesser than 0.2.

**AND - rule**

In the AND fusion rule, as done in the case of OR fusion rule, to achieve a targeted joint probability of false alarm $Q_{f,AND}$ for the network, from (20) the individual secondary users’ targeted probability of false alarm $P_f$ is given by

$$P_f = \frac{M Q_{f,AND}}{\sqrt{M}}$$  \hspace{1cm} (32)
where $M$ is the number of cooperating users. With the $P_f$, the probability of detection $P_d$ of the cooperating users is given by, substituting (32) into (28),

$$P_d = \frac{1}{2} \text{erfc} \left( \sqrt{2} \text{erfc}^{-1} \left( 2 \left( \sqrt{Q_{f,\text{AND}}} \right) \right) - \frac{\sqrt{N \text{SNR}^2}}{2} \right)\right)$$

Substituting this equation into (19) we obtain the joint Probability of Detection.

For fixed number of samples $N=24$ and an average Signal-to-Noise Ratio $\overline{\text{SNR}} = -5 dB$ we obtain the following ROC curves.

![Figure 38: ROC– joint Probability of Detection versus joint Probability of False Alarm (AND fusion rule)](image)

In this figure we can see that the performance improvement of the AND rule is lesser than that of the OR fusion rule. Indeed, we can see that for small $Q_f$, the increase of cooperative users’ number brings a small gain in terms of joint probability of detection. However, when the joint probability of false alarm increases, the performance of cooperative spectrum sensing, based on AND fusion rule, is worst than that of a single user, as shown in the next figure.
The next figure show the behaviour of $Q_d$ and $P_d$ when the number of cooperating users in the network increases.

The previous figure shows that the joint Probability of Detection increases initially and than decreases if the cooperating users’ number increases. Logically, this curves depend on the average SNR and on the individual probability of false alarm. In particular, we considered $\overline{SNR} = -5\, dB$ and $P_f = 0.1$.

Furthermore, the local requirements increase with the number of cooperating users.
In fact, with 10 cooperating users we obtain a joint probability of detection equal to 0.5, but the probability of detection imposed on individual secondary users is close to 0.9.

**MAJORITY - rule**

In the MAJORITY fusion rule that we saw in equations (23) and (24), we compute the joint probabilities of detection and false alarm as functions of $P_d$ and $P_f$ of single user. As said in the previous section these probabilities are the same for all users because we use a fixed average SNR, $\overline{SNR}$.

Unlike the previous fusion rules, the complexity of the equation (24) does not allow an easy computation of $P_f$ as function of joint probability of false alarm $Q_f$. For this reason, to plot ROC curves we compute $P_d$ using equation (28) and then we use directly the equations (23) and (24).

For fixed number of samples $N=24$ and an average Signal-to-Noise Ratio, $\overline{SNR} = -5dB$, we obtain the following ROC curves.

![Figure 41: ROC – joint Probability of Detection versus joint Probability of False Alarm (MAJORITY fusion rule)](image)
In this figure we can see that, as in the OR fusion rule, the performance’s improvement is high if the number of cooperating users is increased. In this way, for a fixed joint probability of false alarm $Q_f$, it’s possible to increase the probability of detection $Q_d$ with an increase of the cooperating users’ number. For example, if we for an operation point ($Q_{f,\text{OR}}=0.1$, $Q_{d,\text{OR}}=0.9$) we have 100% gain in accuracy from 10 users over a single user, as shown in the next figure.

![Figure 42: Gain in accuracy from 10 users over a single user (MAJ rule)](image)

Moreover, the following figure shows that for each value of joint probability of false alarm the joint probability of detection is higher if the number of cooperating users is increased, as seen in the case of OR fusion rule.
4.2.2.2 ROC: Comparison between Fusion Rules

As shown in the previous section, the effect of Cooperative Spectrum Sensing is a performance improvement of the network in terms of $Q_f$ and $Q_d$. This benefits can be viewed in a three manner:

1. **Accuracy Gain**: increased $Q_d$ for fixed $Q_f$;
2. **Less Sophisticated Sensors**: local $P_d$ can be decreased and the system still can achieve the target $Q_d$;
3. **Less Number of Samples**: smaller $P_d$ implies a decrease of the number of sample, as shown by equation (23). Moreover, Sensing time decreases as the number of sample decreases ($T=N/2B$, where $B$ is the bandwidth).

Logically, this depend on the fusion rule. For this reason, in this section we do a comparison between the three fusion rules. Next figure shows ROCs for different fusion rules and 10 collaborating users. As before we use $SNR = -5\,dB$ and $N = 24$ samples. Moreover, to do a detailed analysis we plot also the ROC curve without cooperation, i.e., the case of a single user.
As we can see, there is higher accuracy gain with MAJORITY and OR fusion rules.

4.2.2.3 Complementary Receiver Operation Characteristic

In this section we analyse the joint Probability of MisDetection $Q_m$ as function of joint Probability of False Alarm $Q_f$, hence the Complementary Receiver Operation Characteristic. As done for ROC curves we consider a fixed average Signal-to-Noise Ratio of $\overline{SNR} = -5dB$ and a number of samples $N=24$. Initially, we consider each rule separately and then a comparison between the three rules.

**OR - rule**

In case all secondary users have the same individual $P_{d}$, the joint probability of detection $Q_{d,OR}$ is given by equation (15). Hence, the joint Probability of MisDetection, defined as $(1-Q_d)$, is given by
where $M$ is the number of cooperating users.

As done for ROC curves, to achieve a target joint probability of false alarm $Q_{f,OR}$ for the network, from (16) the individual secondary users’ targeted probability of false alarm $P_f$ is expressed by equation (30). Using $P_f$ of single user, the probability of detection $P_d$ of the cooperating users is given by equation (31). Substituting (31) into (15) we obtain the joint Probability of Detection, and then we can compute the joint probability of misdetection in the case of OR fusion rule using equation (34). Using equation (34) we can plot the Complementary ROC. Result is shown in the next figure.

\[
Q_{m,OR} = 1 - Q_{d,OR} = (1 - P_d)^M
\]

Figure 45: CompROC: joint Probability of MisDetection versus joint Probability of False Alarm (OR rule)

As expected, the joint probability of misdetection assumes very small values when the joint Probability of False Alarm is near to 1. Moreover, $Q_{m,OR}$ decrease when number of cooperating users increase. Hence, the use of collaborative spectrum sensing improve performance in terms of a decreased joint probability of misdetection.
AND - rule

In case all secondary users have the same individual $P_d$, the joint probability of detection $Q_{d,\text{AND}}$ is given by equation (19. Hence, the Probability of MisDetection, defined as $(1-Q_d)$ is given by

$$Q_{m,\text{AND}} = 1 - Q_{d,\text{AND}} = 1 - P_d^M$$

(35)

where $M$ is the number of cooperating users. As done for ROC curves, to achieve a target joint probability of false alarm $Q_{f,\text{AND}}$ for the network, from (20) the individual secondary users’ targeted probability of false alarm $P_f$ is expressed by equation (32). Using $P_f$ of single user, the probability of detection $P_d$ of the cooperating users is given by equation (33). Substituting (33) into (19) we obtain the joint Probability of Detection, and then we can compute the joint probability of misdetection in the case of AND rule using equation (35) and plot the Complementary ROC. Result is shown in the next figure.

![Figure 46: CompROC: joint Probability of MisDetection versus joint Probability of False Alarm (AND rule)](image-url)
As expected from the ROC, the Probability of MisDetection also assumes the same behaviour of the Probability of Detection. Indeed, we can see that for small $Q_f$, the increase of cooperative users’ number brings a small gain in terms of joint probability of misdetection, but when the joint probability of false alarm increases, the performance of cooperative spectrum sensing, based on AND fusion rule, is worst than Local Spectrum Sensing, as shown in the next figure.

![Figure 47: Particular of Comp. ROC (AND rule)](image)

**MAJORITY - rule**

In case all secondary users have the same $P_d$, the joint probability or detection $Q_d,\text{MAJ}$ is given by equation (23). Hence, the Probability of MisDetection, defined as $(1 - Q_d)$, is given by

$$Q_{\text{m,MAJ}} = 1 - Q_{\text{d,MAJ}} = 1 - \sum_{i=0}^{M - \left\lfloor \frac{M}{2} \right\rfloor} \left( \frac{M}{2} \right) \left( 1 - P_d \right)^{M - \frac{M}{2} - i} P_d^{\frac{M}{2} + i}$$

(36)

where $M$ is the number of cooperating users. Unlike the previous fusion rules, the complexity of the equation (24) does not allow an easy computation of $P_f$ as function of joint probability of false alarm $Q_f$. For this reason, to plot Complementary ROC
curves we compute $P_d$ using equation (28) and then we use directly the equations (36) to compute the probability of misdetection for MAJORITY fusion rule. Using equation (36) we can plot the Complementary ROC in the case of MAJORITY Fusion rule. Result is shown in the following figure.

![Figure 48: CompROC: joint Probability of MisDetection versus joint Probability of False Alarm (MAJORITY rule)](image)

As expected, the joint probability of misdetection $Q_m$ decreases when number of cooperating users increases. Hence, the use of collaborative spectrum sensing improve performance in terms of a decreased joint probability of misdetection.

### 4.2.2.4 CROC: Comparison between Fusion Rules

As done for the ROC, in this section we do a comparison for Complementary ROC between the three fusion rules. Next figure shows Complementary ROC for different fusion rules and for 10 collaborating users. As before we use $\text{SNR} = -5dB$ and $N = 24$ samples. Moreover, to do a detailed analysis we plot also the ROC curve without cooperation, i.e., the case of a single user.
Figure 49: Comp. ROC curves – Comparison between Fusion Rules

As expected from Complementary ROCs, there is higher performance’s improvement with OR and MAJORITY fusion rules.

4.2.3 Behaviour in Average SNR

The goal of this subsection is to analyze the performances of Cooperative Spectrum Sensing as function of Signal-to-Noise Ratio. The analysis is divided in two parts. The first involves the Probability of Detection while the second involves the Probability of MisDetection. For each part we consider the OR fusion rule. We consider the following range of average SNR:

- **Medium Range:** \(-5 \leq \text{SNR} \leq 5\)

Moreover, we fixed the number of samples at the same value of the previous section \((N = 24)\) and the joint probability of false alarm to \(Q_f = 0.1\).
4.2.3.1 Probability of Detection versus Signal to Noise Ratio

We want analyse the Probability of Detection for the network \( Q_d \) as function of average SNR for a variable number of cooperating users.

For a number of users \( n = [1,2,3,4,5,10] \) we obtain the following curves.

As expected, the plot shows that the joint probability of detection increases with the number of cooperating users. Results indicate a significant improvement in term of required average SNR for detection. In particular, to achieve a probability of detection equal to 0.9, local spectrum sensing requires SNR=0 dB while collaborative sensing with \( n = 10 \) only needs a average SNR of -3.68 dB for individual users, as shown in the following figure.
Hence, distributed spectrum sensing yields a 3.68 dB performance gain over local spectrum sensing for the operation point \((Q_f, Q_d) = (0.1, 0.9)\) with only 10 users.

### 4.2.3.2 Probability of MisDetection versus Signal to Noise Ratio

In this section we analyse the joint probability of misdetection \(Q_m\) as function of average SNR for a variable number of cooperating users.

For a number of users \(n = [1,2,3,4,5,10]\) we obtain the following curves.
Figure 52: joint Probability of MisDetection ($Q_m$) versus SNR (OR fusion rule)

The plot shows that the Probability of MisDetection decreases as the number of cooperating users increases. Results indicate a significant reduction in term of required average SNR for misdetection. In particular, for a probability of misdetection equal to 0.1, local spectrum sensing requires SNR=0 dB while collaborative sensing with $n=10$ only needs an average SNR of -3.68 dB for individual users, as shown in the following figure.

Figure 53: performance gain in SNR (DSS – $Q_m$ - OR rule)
4.3 Conclusions

This sections explored the improvements in spectrum sensing performance achievable through network cooperation and showed that sensing reliability improves monotonically with the number of cooperative radios in the case of OR and MAJORITY fusion rules. However, in practical network scenarios cooperation requires sharing information among cognitive radios. In particular, secondary users’ cooperation requires to share and coordinate individual sensing decision. Hence, cooperation requires a common control channel with minimized load, and low latency which increases if the number of cooperative users is increased. Moreover, control channel requires a physical communication channel which occupies spectrum resources, thus its load should be minimized.
Chapter 5

LOCAL SPECTRUM SENSING USING SINGLE-STAGE SENSING

In Cognitive Radio systems that use Dynamic Frequency Selection (DFS), spectrum sensing is essential for the protection of legacy spectrum users. In particular, in the absence of cooperation or signalling between the primary licensee and the secondary user, spectrum availability for the opportunistic spectrum access may be determined by direct spectrum sensing. Moreover, in the normal operation mode, the secondary user has to detect the channel periodically during its data transmission to decide whether the channel is idle. Hence, in order to avoid unacceptable interferences to primary user, the secondary user must follow the next two principles:

- Detect the channel before starting data transmission to decide whether it is idle. A high probability of detection has to be achieved;
- Detect the channel periodically during data transmission.

Assuming that the secondary user has detected an idle channel before data transmission, we consider only the second principle that it is known as In Band Spectrum Sensing. After detecting a white space, the secondary user starts utilizing it by properly tuning its transmission parameters. However, secondary user should periodically sense the licensed spectrum in case a primary user starts to transmit.

In this section we investigate the optimum allocation of the sensing time according with the requirements about the sensing accuracy defined in the IEEE 802.22 standard.
5.1 Overview on Spectrum Sensing using Single-Stage Sensing

In this section, we describe the basic definitions of the Local Spectrum Sensing (LSS) using Single Stage Sensing (SSS). To do this, we consider the simplest cognitive radio system model which has only one available channel and a pair of licensed user and secondary user. As usual, the primary user does not always occupy the channel, which leads the channel being underutilized in the time domain. The secondary user is the cognitive radio user which is permitted to use the channel only when the licensed user is absent (Opportunistic Spectrum Access or Spectrum Overlay). To introduce LSS–SSS consider the following figure in which *Intra-frame Sensing* is shown.

![Frame structure for Cognitive Radio Networks with periodic spectrum Sensing](image)

As specified in the IEEE 802.22 WRANs standard draft, the final decision must be made before the *Channel Detection Time* (*CDT*) that is defined as the maximum time for the sensing device to decide on the channel status. This means that sensing device must ensure at least one chance to sense the channels in a *CDT* interval.

Let $FS$ and $L$ represent the size of an IEEE 802.22 frame (10 msec) and the duration of a quiet period, respectively. The parameter $L$ depends on the number of samples collected and it’s the time needed by the detection technique to sample the channel. The 802.22 MAC allows only one intra-frame quiet period per frame and it must be scheduled always at the end of the frame. Moreover, the sensing device decides whether to schedule an intra-frame quiet period over multiple frames in order to
perform more detailed sensing. Hence, the most important settings concern the allocation of quiet periods and its duration. Since consecutive quiet periods are carried out, we assume that they are allocated in a periodic manner. We represent the periodic allocation of SSS using a *Sensing Period* \((T)\)

\[
T = \alpha \cdot FS, \tag{1}
\]

where \(\alpha \in \left[1, \left\lfloor \frac{CDT}{FS} \right\rfloor \right]\) is the *allocation coefficient*. The duration of a quiet period must be shorter than the sensing period

\[
L < T = \alpha \cdot FS. \tag{2}
\]

The Single Stage Sensing’s structure is show in the next figure

![Figure 55: Single Stage Sensing based on DFS time requirements for IEEE 802.22 WRANs](image)

Moreover, the number of quiet periods of SSS is given by

\[
M = \left\lfloor \frac{CDT}{T} \right\rfloor. \tag{3}
\]

Both allocation coefficient and quiet period’s duration must be set in a combined manner in order to obtain a good sensing accuracy and a high throughput.
5.2 Single Slot Spectrum Sensing

In this subsection we analyse the Local Spectrum Sensing using Single Stage Sensing (LSS-SSS). In this case there is not cooperation between sensing devices and each secondary user uses an allocation coefficient equal to 200. Hence, the sensing period is equal to 2 seconds. In other words, we consider the simple case of a Single Slot in a CDT interval, as shown in the next figure.

![Diagram of Single Slot Spectrum Sensing using Single Stage Sensing](image)

Figure 56: Single Slot Spectrum Sensing using Single Stage Sensing

5.2.1 Sensing Accuracy

As seen in the Chapter 4, the performance analysis of the spectrum sensing can be done using the probabilities of false alarm and detection

\[ P_f = P(H_0 | H_1) = Q\left(\frac{\gamma - N\sigma_v^2}{\sigma_v \sqrt{2N}}\right), \]  

(4)
$$P_d = P(H_1 | H_1) = Q \left( \frac{\gamma - N \sigma_v^2 - Np_s}{\sigma_v \sqrt{2N \sigma_v^2 + 4Np_s}} \right), \quad (5)$$

respectively. As done in the Chapter 4, it’s easy to see that the two previous probabilities are given by

$$P_f = Q \left( Q^{-1}(P_d) \sqrt{1 + 2SNR} + \sqrt{\frac{L \cdot B \cdot SNR^2}{2}} \right) \quad (6)$$

$$P_d = Q \left( \frac{Q^{-1}(P_f) - \sqrt{\frac{L \cdot B \cdot SNR^2}{2}}}{\sqrt{1 + 2SNR}} \right) \quad (7)$$

where $B$ is the channel bandwidth and $L$ is the duration of the quiet period. Moreover, in order to obtain a high sensing accuracy in SSS, we must minimize the probability that the sensing device declares the channel as idle when it is actually busy, $P(H_0 | H_1)$, which measures the probability that a secondary users cause harmful interference to the primary user (Probability of MisDetection)

$$P_{md} = 1 - P_d = 1 - Q \left( \frac{Q^{-1}(P_f) - \sqrt{\frac{L \cdot B \cdot SNR^2}{2}}}{\sqrt{1 + 2SNR}} \right), \quad (8)$$

As we can see from equation (6), (7) and (8), the sensing accuracy depend on the number of samples collected. Using these equations, we can analyse the behaviour of the sensing accuracy as function of the quiet period’s duration.

From equation (2), we can see that from the regulator’s viewpoint it suffices for the secondary system to be able to monitor the band and make a decision about the presence of the licensee once in every $T$ seconds. However, from the secondary
user’s point of view, it is desired to maintain the time $L$ required for sensing well below $T$ in order to maximize the time available for data transmission. In other hands, while choosing a smaller $L$ allows to obtained a higher channel utilization, it results in a higher probability of false alarm. This is an important aspect because the increased false alarm result in a higher number of unnecessary channel evacuations and channel-search periods (the time used to search a new idle channel), thereby reducing the average channel utilization.

### 5.2.2 Channel Utilization

In the previous section we have seen that the sensing accuracy of LSS-SSS can be characterized using equations (6) and (7). Logically, we obtain a higher sensing accuracy if the duration $L$ of quiet period is increased. This yields however the data throughput achievable in to decrease. For this reason, we must find a measure for data throughput (a suitable duration of quiet periods) in order to balance these two contrasting quantities. In this analysis we use the Channel Utilization $\rho$ to characterize the system.

Before to start our analysis, the following considerations are necessary. In the normal operation mode, the secondary user transmits its information, senses the environments periodically and makes a final decision about the state of the channel. When the secondary user declares the channel as busy, it must search another channel where continue its transmission.

To analyse the channel utilization, we divide our work in two part:

1. **In-Band Channel Utilization**: the channel is declared idle and sensing device is in Normal Operation Mode;
2. **Average Channel Utilization**: this is the general scenario in which the channel can be declared idle or busy.

Moreover, as done for the sensing accuracy we consider the point of view of the secondary user in which we analyse the behaviour of the channel utilization as
function of the sensing time in order to find the optimum sensing time that maximize it. Hence, the goal of this subsection is to investigate the trade off between the channel utilization and sensing time.

5.2.2.1 In-Band Channel Utilization

Let the hypotheses $H_0$ and $H_1$ occur with probabilities $P(H_0) = P_{idle}$ and $P(H_1) = P_{busy} = 1 - P_{idle}$, respectively. Then, during normal operation mode, sensing device decide for an idle channel in the two following cases:

1. Channel idle and declared idle: this is a correct decision about the idle state of the channel. Its probability is equal to $P(H_0 \mid H_0) = P_{idle} \cdot (1 - P_f)$;

2. Channel busy and declared idle: this is the case of misdetection of the primary user. Its probability is equal to $P(H_0 \mid H_1) = P_{busy} \cdot (1 - P_d)$.

Hence, the probability that the channel is declared idle is given by

$$P_s = (1 - P_f) \cdot P_{idle} + (1 - P_d) \cdot P_{busy}.$$  \hspace{1cm} (9)

Let $(T - L)/T$ the percentage of time used for data transmission, the channel utilizations of the two previous cases can be defined as follow

$$\rho_1 = [(1 - P_f) \cdot P_{idle}] \left(\frac{T - L}{T}\right),$$  \hspace{1cm} (10)

$$\rho_2 = [(1 - P_d) \cdot P_{busy}] \left(\frac{T - L}{T}\right),$$  \hspace{1cm} (11)

respectively. Hence, the In-Band Channel Utilization can be defined as follow
\[ \rho_{IB} = \rho_1 + \rho_2 = \left[ (1 - P_f) \cdot P_{idle} + (1 - P_d) \cdot P_{busy} \right] \left( \frac{T - L}{T} \right), \]  

(12)

which is the ratio of the amount of time available for data transmission to the total amount of time the channel remains perceived idle. Using equation (9), the In-Band Channel Utilization can be expressed as follows:

\[ \rho_{IB} = P_s \left( \frac{T - L}{T} \right). \]  

(13)

### 5.2.2.2 Average Channel-Search Time

In this subsection, we analyse the impact of the Average Channel-Search Time \( T_{search} \). Let \( T^* \) denote the time spent for sensing each channel using energy detection, according with the performance requirements about the probabilities of false alarm and detection, during the channel-search period. For a given probability of false alarm \( P_f \) and probability of detection \( P_d \) during the search period, the probability that a channel is declared idle and is acquired for the secondary transmission, is given by

\[ P_s = (1 - P_f) P_{idle} + (1 - P_d) P_{busy}, \]  

(14)

where \( P_{idle} \) and \( P_{busy} \) be the probabilities of the hypotheses \( H_0 \) and \( H_1 \), respectively. The first term in (14) corresponds to the successful identification of a white space, while the second term represents the case where the channel is falsely deemed idle due to the non-detection of the primary signal. Since we suppose to have an high probability of detection during the channel-search, the second term is negligible and we may approximate \( P_s \) in the following manner.
\[ P_s \approx (1 - P_f)P_{idle}. \]  

Assuming that the number of primary channel, \( C \), is to be sufficiently large such that during the channel-search, with high probability, at least one of the primary channels is idle, that is

\[ P_{\text{busy}}^{C-1} \leq (1 - P_s)^{C-1} \ll 1, \]  

as demonstrated in [1], the average time to find an idle channel is given by

\[
\bar{T}_{\text{search}} = P_s T^s \sum_{k=1}^{C-1} k(1 - P_s)^{k-1} = T^s \left[ \frac{1 - (1 - P_s)^C}{P_s} - C(1 - P_s)^{C-1} \right] \approx \frac{T^s}{P_s}, \quad (17)
\]

where the last approximation in (17) follows from the high probability that at least one channel is declared idle \((1 - P_s)^{C-1} \ll 1\). Substituting (15) into (17), we obtain

\[
\bar{T}_{\text{search}} = \frac{T^s}{(1 - P_f)P_{idle}}. \quad (18)
\]

Hence, the optimum Sensing Time for each channel can be formulated as follow:

\[
\hat{T}^s = \arg \min_{T^s} \frac{T^s}{(1 - P_f)P_{idle}} \quad \text{subject to} \quad T^s \geq T_{\text{min}}^s
\]

\[
P_f = Q \left( Q^{-1}(P_d) \sqrt{1 + 2SNR} + \frac{T_s \cdot B \cdot SNR^2}{2} \right)
\]

where \( T_{\text{min}}^s \) is the minimum sensing time to obtain the maximum value of the probability of false alarm. It can be computed using equation (6) as follow
\[ T_{\text{min}}^S = \frac{2}{B} \left( \frac{Q^{-1}(P_f) - Q^{-1}(P_d)\sqrt{1 + 2\text{SNR}}}{\text{SNR}} \right), \] (20)

where \( L \) was replaced with the minimum sensing time \( T_{\text{min}}^S \).

The trade off between the channel-search time and the quality of sensing can be observed plotting the average channel-search time as function of the sensing, as shown in the next figure, in which we consider a probability of detection equal to 0.99, a SNR equal to -20.8 dB and a bandwidth equal to 6 MHz. In particular, when the sensing time for each channel increases, the \( T_{\text{search}} \) is decreased. However, beyond a certain point, this gain is outweighed by the long sensing times.

Figure 57: Single Slot LSS - Average Channel-Search Time VS Sensing Time (\( P_d=0.99 \))
In the previous figure we can see the same behaviour of the average channel search when it is function of the probability of false alarm and for an underutilization of the channel equal to 0.15. In particular, in this case we obtain a sensing time for each channel equal to 21.94 ms, a probability of false alarm equal to 0.58 and a average channel-search time equal to 105.43 ms. Logically, this values depend on the underutilization of the channel, hence, on the probability of the hypothesis $H_0$. In the next table are shown the optimum values $\bar{T}_{\text{search}}$ for three different underutilization of the channel.

<table>
<thead>
<tr>
<th>$P(H_0)$</th>
<th>$\bar{T}_{\text{search}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>351.45 ms</td>
</tr>
<tr>
<td>0.50</td>
<td>105.43 ms</td>
</tr>
<tr>
<td>0.85</td>
<td>62.02 ms</td>
</tr>
</tbody>
</table>

Table 2: Single Slot LSS - Optimum values of the Average channel-Search Time

To conclude this subsection, in the next figures we plot again the average search-time as function of sensing time and of the probability of false alarm for a different value of the target probability of detection ($P_d = 0.999$).
Figure 59: Single Slot LSS - Average Channel-Search Time VS Sensing Time ($P_d = 0.999$)

Figure 60: Single Slot LSS - Average Channel-Search Time VS Probability of False Alarm

($P_d = 0.999$ and $P_{idle} = 0.15$)
5.2.2.3 Average Channel Utilization

As seen in the previous section, the secondary user must search a new idle channel only when it decides for the hypothesis $H_1$. In this subsection, we analyse the channel utilization when the sensing device declares the channel. A channel is declared busy in the two follows cases:

1. **Channel busy and declared busy**: this is a correct detection of the primary user’s transmission. Its probability is equal to
   \[
   P(H_1 | H_1) = P(H_1) \cdot P_d = P_{busy} \cdot P_d
   \]

2. **Channel idle and declared busy**: this is the case of false alarm decision about the state of the channel. Its probability is equal to
   \[
   P(H_1 | H_0) = P(H_0) \cdot P_f = P_{idle} \cdot P_f
   \]

The probability that a channel is declared busy is given by

\[
P_e = P_{idle} \cdot P_f + P_{busy} \cdot P_d
\]  \quad (21)

In this case, the sensing device must cease all interfering transmission on the current channel, search a new idle channel and start again the transmission. Moreover, if hypothesis $H_1$ is occurred the sensing device, or the Base Station, shall modify the system operating parameters for the new transmission. Hence, the total time from the detect of the primary user and the new transmission are the following:

1. **Channel Move Time** ($T_{Move} = 2 \text{ sec}$): the time taken by a WRAN system to cease all interfering transmissions on the current TV channel upon detection of a licensed incumbent signal above the relevant Incumbent Detection Threshold;
2. **Channel Setup Time** ($T_{\text{Setup}} = 2$ sec): the window of time that may be taken by a WRAN CPE to transmit control information to a WRAN base station in order to establish operation with that base station at the prescribed power.

The total time used by the sensing device to transmit, sense the channel, search a new channel and start new transmission is equal to $T_{\text{tot}} = T + T_{\text{search}} + T_{\text{Move}} + T_{\text{Setup}}$. Hence, using equations (13) and (21), the Average Channel Utilization can be defined as follow

$$\rho = P_s \left( \frac{T - L}{T} \right) + P_e \left( \frac{T - L}{T_{\text{tot}}} \right).$$  \hspace{1cm} (22)

Substituting $P_s$ and $P_e$ using equations (9) and (21), we obtain

$$\rho = \left[ (1 - P_f)P_{\text{idle}} + (1 - P_d)P_{\text{busy}} \right] \left( \frac{T - L}{T} \right) + \left[ P_f P_{\text{idle}} + P_d P_{\text{busy}} \right] \left( \frac{T - L}{T_{\text{tot}}} \right).$$  \hspace{1cm} (23)

### 5.2.3 Cognitive Radio Transmission Scenarios

As we seen in the previous subsections, from the regulator’s viewpoint it suffices for the secondary system to be able to monitor the band and make a decision about the presence of the licensee once in every $T$ seconds. Based on this consideration, we can define two different scenarios:

1. **Constant Primary User Protection (CPUP)**;
2. **Constant Secondary User Spectrum Utilization (CSUSU)**.

In the following subsection, we analyse the behaviour of sensing accuracy, the In-Band Channel Utilization and the Average Channel Utilization as function of the...
sensing time. The goal of this section is to see if the radio transmission scenarios
defined above it’s possible to set an optimum sensing time in order to guaranteed the
sensing accuracy defined in the DFS timing requirements that maximizes the channel
utilization.

5.2.3.1 Constant Primary User Protection

The first transmission mode is viewed from the primary user’s point of view and it
 guarantees a minimum level of interference. This scenario can be realized by fixing
the probability of detection at the required level $\bar{P}_d$ and minimizing the probability
of false alarm in (6) as much as possible

$$P_f = Q \left( Q^{-1} \left( \bar{P}_d \right) \sqrt{1 + 2SNR} + \sqrt{\frac{L \cdot B \cdot SNR^2}{2}} \right). \quad (24)$$

In other words, the probability of false alarm can be minimized by increasing the
quiet period’s duration.

According with the DFS requirements, we consider the incumbent detection
threshold of the DTV signal ($P_s = -116 \text{ dBm}$) and a noise level equal to
$P_n = -95.2 \text{ dBm}$ (bandwidth equal to 6 MHz and $NF = 11 \text{ dB}$).

In the following subsection, we analyse the behaviour of the probability of false
alarm, the In-Band Channel Utilization and the Average Channel Utilization as
function of the sensing time.
As we can see, at the same sensing time, increasing the primary user’s protection level by stating higher probability of detection values leads to increase the probability of false alarm and consequently, fewer chances for the secondary user to utilize the channel. Therefore, there will be a trade off between these two conflicting objectives. This trade off can be analyzed plotting the channel utilization as function of the sensing time.

In the CPUP scenario we consider a target probability of detection \( \overline{P}_d \) in order to guarantee a fixed protection of the primary user. In this case, using equation (12) the In-Band Channel Utilization can be express as follow

\[
\rho_{IB} = \left[ (1 - P_f) \cdot P_{idle} + (1 - \overline{P}_d) \cdot P_{busy} \right] \frac{T - L}{T}.
\]  

(25)

In practice, the target probability of detection is chosen to be close to but less than 1. In particular, in IEEE 802.22 WRAN the requirements establish the minimum value equal to 0.9. Since we suppose that the activity probability of the primary user is small, say less than 0.5, we can neglect \( \rho_2 \). In fact, studies have shown that the
utilization of the channels ranging from 0.15 to 0.85, depending on the location and time of the day. In the worst case in which $P_{\text{busy}} = 0.85$, the probability that a busy channel is declared idle by the secondary user is equal to 0.085. Hence, the In-Band Channel Utilization can be approximate as follow

$$\tilde{\rho}_{IB} = \left[1 - P_f \cdot P_{\text{idle}}\right] \left(\frac{T - L}{T}\right).$$  \hspace{1cm} (26)

To justify our approximation in (26), in the next figure we do a comparison between the effective In-Band Channel Utilizations and its approximation.

![Figure 62: LSS-SSS - Single Slot Sensing – Comparison between Approximate and Effective In-Band Channel Utilization (Pidle=0.85 - CPUP)](image)

The behaviour of the In-Band Channel Utilization as function of the sensing time is shown in the following figures.
Figure 63: LSS-SSS - Single Slot Sensing – Effective Channel utilization versus Sensing Time (CPUP)

Figure 64: LSS-SSS - Single Slot Sensing – Approximate Channel utilization versus Sensing Time (CPUP)

As we can see from the previous figures, increasing the sensing accuracy by sensing each channel for a longer time, reduces the probability to detect the a busy channel thanks to the decreased false alarm. Beyond a certain point, however, this gain is
outweighed by the longer sensing time. This means that the parameter \( L \) must be chosen in order to maximize equation (25). Hence, in a CPUP scenario, the optimum sensing time \( L_{\text{opt}} \) for channel utilization in normal operation mode must be formulated as follows:

\[
L_{\text{opt}} = \arg \max_L \left\{ \left(1 - P_f\right) \cdot P_{\text{idle}} + \left(1 - \overline{P}_d\right) \cdot P_{\text{busy}} \left( \frac{T - L}{T} \right) \right\}.
\]

(27)

\[
P_f = Q\left(\Phi^{-1}(\overline{P}_d) \sqrt{1 + 2\text{SNR}} + \sqrt{\frac{L \cdot B \cdot \text{SNR}^2}{2}}\right)
\]

\[
\text{Approx In-Band Channel Utilization}
\]

\[
\text{Average Channel Utilization 0.83}
\]

\[
\text{Optimum Sensing Time 68.13 ms}
\]

![Graph showing channel utilization and sensing time](image)

Figure 65: Single Slot Sensing – Optimum Sensing Time (In-Band Channel Utilization - CPUP)

In particular, for the system value defined above, the Optimum Sensing Time is equal to 68.13 ms. Using this value we obtain an In-Band Channel Utilization equal to 0.83 and a \( P_f = 0.0068 \).

Now, we consider the behaviour of the Average Channel Utilization using equation (23), that in CPUP scenario can be express as follow:
\[
\rho = \left(1 - P_f\right)P_{idle} + \left(1 - \bar{P}_d\right)P_{busy}\left(\frac{T - L}{T}\right) + \left[P_f P_{idle} + \bar{P}_d P_{busy}\right]\left(\frac{T - L}{T_{tot}}\right). \quad (28)
\]

Considering the comments about the probability of detection and its influence to the two channel utilization above, we can approximate the effective Average Channel Utilization as follow

\[
\tilde{\rho} = \left(1 - P_f\right) \cdot P_{idle}\left(\frac{T - L}{T}\right) + \left[P_f \cdot P_{idle} + P_{busy}\right]\left(\frac{T - L}{T_{tot}}\right). \quad (29)
\]

To justify our approximation in (29), in the next figure we do a comparison between the effective Average Channel Utilizations and its approximation. In the following plot we consider a \( P_{idle} = 0.85 \), that represents practice value.

As we can see, the approximate average channel utilization in (29) is close to the effective average channel utilization in (28). Plots of the Average Channel Utilization as function of the sensing time are shown in the next figures.
The previous figures show that decreasing the probability of false alarm does not lead to an absolute increase of the secondary user throughput as thought but instead,
there is an optimal sensing time at which the channel utilization is maximize. Moreover, the figure also reveals that the channel utilization increases if the underutilization of the channel \( P_{idle} \) is increased. Hence, we conclude that the parameter \( L \) must be chosen in order to maximize the equation (28). Hence, in a CPUP scenario, the optimum sensing time \( L_{opt} \) for channel utilization must be formulate as follow

\[
L_{opt} = \arg \max_L \left\{ \left[ (1 - P_f)P_{idle} + (1 - P_d)P_{busy} \right] \left( \frac{T - L}{T} \right) + \left[ P_f P_{idle} + P_d P_{busy} \right] \left( \frac{T - L}{T_{tot}} \right) \right\}
\]

\[
P_f(P_d, L) = \sqrt{Q^{-1}(P_d)} \sqrt{1 + 2SNR} + \sqrt{\frac{L \cdot B \cdot SNR^2}{2}}
\]

Figure 69: Single Slot Sensing –Optimum Sensing Time (Average Channel Utilization - CPUP)

In particular, for the system value defined above, the Optimum Sensing Time is equal to 60.1 ms. Using this value we obtain an Average Channel Utilization equal to 0.875 and a \( P_f = 0.012 \). This value of \( P_f \) is lesser than the maximum value defined by the Functional Requirements for the 802.22 WRAN Standard \( (P_f,_{max} = 0.1) \).
The simulation results shown that, in local single slot spectrum sensing under CPUP transmission mode, the maximum secondary user’s capacity is achieved at a unique optimum sensing time, which can be found maximizing equation (30).

5.2.3.2 Constant Secondary User Spectrum Utilization

The second scenario is taken from the secondary user’s perspective; it aims to standardize the spectrum utilization by secondary user. As such, the $P_f$ values should be fixed at lower values, according to the standard’s requirements, while keep maximizing $P_d$ which can be written using (7) as follows

$$P_d = Q\left[\frac{Q^{-1}(P_f) - \sqrt{\frac{L \cdot B \cdot SNR^2}{2}}}{\sqrt{1 + 2SNR}}\right].$$

Increasing the sensing time leads to an improvement on the primary user’s protection represented by increasing the probability of detection. In the following figure, we plot the behaviour of the probability of detection as function of the sensing time. As done in the CPUP scenario, according with the DFS requirements, we consider the incumbent detection threshold of the DTV signal ($P_s = -116 dBm$) and a noise level equal to $P_n = -95.2 dBm$ (bandwidth equal to 6 MHz and $NF = 11 dB$).
As we can see, at the same sensing time, increasing the spectrum usability by decreasing the probability of false alarm leads to decrease $P_d$ that is the protection of primary user. Again, there will be a trade off between these two conflicting objectives. In the CSUSU scenario, we consider a target probability of detection $P_f$ in order to guarantee a fixed spectrum utilization of the secondary user. In this case, using equation (12) the In-Band Channel Utilization can be express as follow

$$
\rho_{IB} = \left[ (1-P_f) \cdot P_{idle} + (1-P_d) \cdot P_{busy} \right] \left( \frac{T-L}{T} \right).
$$

(32)

Since in this scenario the probability of detection depend on the time used to sense the environment, the probability that sensing device declare the channel idle when it is busy is not negligible. Moreover, according with the requirements about the sensing accuracy, for this scenario we consider the maximum value allowed for the probability of false alarm, $P_f = 0.1$. 

Figure 70: LSS-SSS - Single Slot Sensing - Probability of Detection versus Sensing Time (CSUSU)
As we can see in the previous figure, the In-Band Channel Utilization decreases with increasing the sensing time as well as increasing the protection level of the primary user. Now, from equation (23) the Average Channel Utilization for CSUSU scenario can be expressed as follows:

\[
\rho = \left[ (1 - P_f) \rho_{idle} + (1 - P_d) \rho_{busy} \right] \left( \frac{T - L}{T} \right) + \left[ P_f \rho_{idle} + P_d \rho_{busy} \right] \left( \frac{T - L}{T_{tot}} \right)
\]  

(33)

Again, since in this scenario the probability of detection depends on the time used to sense the environment, the probability of detection is not negligible and, according with the requirements about the sensing accuracy, for this scenario we consider the maximum value allowed for the probability of false alarm, \( P_f = 0.1 \).

The result is shown in the next figure.
Again, the average channel utilization decreases if the sensing time is increased.

To conclude this subsection, we can say that, in local single slot spectrum sensing under CSUSU mode, there is no optimal sensing time at which the secondary user capacity can be maximized. The secondary user capacity continuously to decrease with increasing the sensing time as well as increasing the protection level of the primary user. Hence, the “Optimum Sensing Time” must be chosen in order to guaranteed the minimum value of primary user’s protection. In particular, according with the requirements about sensing accuracy, sensing time must be chosen in order to guarantee a probability of detection equal to 0.9, as shown in the next figure.
Using this Sensing time, that is equal to 32 ms, we can compute the maximum Average Channel Utilization of the secondary user. In particular, in the following plot we can see that in the case of CSUSU scenario it is equal to 0.839.
5.2.4 Comparison between CPUP and CSUSU Scenarios

As we have see in the previous subsection, the selection of the optimum sensing time is made based on the scenario. In particular, in the CPUP scenario the optimum sensing time is select according with the maximization problem in (30), while in the CSUSU scenario it is the value that allowed to obtain the minimum value for the probability of detection equal to 0.9.

The goal of this section is to understand in which scenario we obtain the higher Average Channel Utilization. To do this, we consider the following figure in which the Average Channel Utilization as function of the sensing time is shown.

![Comparison between CPUP and CSUSU Scenarios (Channel Utilization)](image)

As we can see, the values of the Average Channel Utilization are equal to 0.875 in the CPUP Scenario and 0.839 in the CSUSU Scenario. Hence, if we must chose between the two transmission mode, the CPUP is better than CSUSU, but this difference is very small.
5.3 Multi Slot Spectrum Sensing

In this section, we analyze the case in which the number of the quite period is largest than one. Hence, the goal of this section is to analyse if using the multi slot spectrum sensing we obtain an increase of the channel utilization, and so a decrease of the sensing time. To do this, we introduce a new variable in our analysis: the allocation coefficient $\alpha$. As seen in equations (1) and (3), both the sensing period and the number of quiet periods depend on the value of $\alpha$, respectively. Hence, the single slot in the previous section is split into multiple discontinuous mini-slot. Let $L_i$ be the duration of a quiet period for each slot, and again $M$ and $T$ the number of quiet period (number of mini-slot) and the sensing period, respectively. Without loss of generality, we fix the total sensing time to $L = L_i M$, and the number of samples equal to $N_i = N/M = LB/M$. This means that the quiet periods’ duration is the same for each mini-slot. At the end of the $CDT$ interval, the final decision is based on the fusion of the samples collected during the $M$ quiet periods. The Multi Slot Single Stage Sensing’s structure is shown in the next figure.

![Diagram](image-url)
As demonstrated in [2], according with the our hypothesis of energy detection

\[ H_0 : x_i(n) = v_i(n) \]

\[ H_1 : x_i(n) = h_i s(n) + v_i(n) \]

\[ n = 1, 2, \ldots, N, \quad (34) \]

in which we consider the case of static channel \((h_i = 1)\), the use of \(M\) mini-slot does not provide any performance gain when data fusing is applied. For this reason, in our work we consider only the case of Decision Fusion, in which the final decision at the end of the CDT interval is based on the \(M\) decision made during the \(M\) quiet periods, as shown in the next figure.

![Decision Fusion Diagram](image)

**Figure 77: LSS - Multi Slot Decision Fusion**

Assuming that all decision are independent, and supposing that

\[ P_d^{(1)} = \ldots = P_d^{(M)} = P_{d,0} \quad \text{and} \quad P_f^{(1)} = \ldots = P_f^{(M)} = P_{f,0}, \]

where the probabilities of false alarm and detection for each quiet periods is given by equations (6) and (7), in this
subsection we analyse the impact of this model on the performance of spectrum sensing using different fusion rules. In particular, under the hypothesis that the primary user either active or inactive for all the $M$ mini-slots, the probabilities of detection $P_d$ and false alarm $P_f$ in multi slot spectrum sensing can are given by

- **“LOGIC OR” fusion rule:** in the final decision sensing device decides that the primary user is present if at least in one mini slot the decision $H_1$ is made. The probabilities of detection $P_{d,OR}$ and false alarm $P_{f,OR}$ can therefore be given as

\[
P_{d,OR} = 1 - \left(1 - P_{d,0}\right)^M
\]

\[
P_{f,OR} = 1 - \left(1 - P_{f,0}\right)^M
\]

The average channel utilization can be express using equation (22), where the two probabilities, $P_s$ and $P_e$, are given by

\[
P_s = (1 - P_{f,OR})P_{idle} + (1 - P_{d,OR})P_{busy}
\]

\[
P_e = P_{f,OR}P_{idle} + P_{d,OR}P_{busy}
\]

- **“LOGIC AND” fusion rule:** in the final decision sensing device decides that the primary user is present only if in all mini the decision $H_1$ is made. The probabilities of detection $P_{d,AND}$ and false alarm $P_{f,AND}$ are given by

\[
P_{d,AND} = P_{d,0}^M
\]

\[
P_{f,AND} = P_{f,0}^M
\]
The average channel utilization can be express using equation (22), where the two probabilities, $P_s$ and $P_e$, are given by

\[ P_s = (1 - P_f, AND)P_{idle} + (1 - P_d, AND)P_{busy} \]  
\[ P_e = P_f, ANDP_{idle} + P_d, ANDP_{busy} \]  

### 5.3.1 Decision Fusion under CPUP Scenario

In this subsection, considering both the Logic OR and Logic AND fusion rules, we analyse the performance of Decision Fusion in multi slot spectrum sensing under the Constant Primary User Protection Scenario.

As done in the case of single slot, we use a target Probability of Detection of the final decision equal to 0.9, and, according with the DFS requirements, we consider the incumbent detection threshold of the DTV signal ($P_s = -116 \text{ dBm}$) and a noise level equal to $P_n = -95.2 \text{ dBm}$ (bandwidth equal to 6 MHz and $NF = 11 \text{ dB}$).

#### 5.3.1.1 Logic OR Fusion Rule

Using equations (22), (37) and (38), we analysis the Average Channel Utilization as function of the sensing time $L$ when Logic OR fusion rule is adopted.

As we can see in the next figure, the use of multi slot spectrum sensing with Logic OR Fusion Rule doesn’t increase the performance in term of channel utilization.
This decreased of performance is due to the behaviour of the Probability of False Alarm $P_{f,OR}$, that is plot in the next figure.
As we can see, the probability of false alarm decreases if the sensing time $L$ is increased. Since the probability of false alarm increases if the number of quiet periods is increased, the average channel utilization decrease. In fact, in the CPUP scenario, it depend only on the $P_{f,OR}$ and $L$. Hence, for a fixed sensing time, the decrease of the channel utilization is due to the increase of the probability of false alarm.

Now, we consider the following plot in which the probabilities of False Alarm, of the final decision and the single slot, and the Probability of Detection of the single slot are shown.

![Multi Slot Sensing - Probabilities of False Alarm and Probability of Detection (Single Slot) versus M (Logic OR Fusion Rule – $P_d=0.9$, $P_{idle}=0.85$, $L=10$ ms)](image)

As we can see, for a fixed probability of detection $P_{d,OR}$, the probabilities of false alarm and detection of single slot, $P_{f,0}$ and $P_{d,0}$, respectively, decrease if the number of quiet periods is decreased. Hence, for a fixed $P_{d,OR}$, if the number of quiet periods increases, the requirements about $P_{d,0}$ can be relaxed and, moreover, since for smaller probability of detection of single slot the threshold increases, the probability of false alarm, $P_{f,0}$, decreases. On the contrary, the probability of false
alarm $P_{f,OR}$ increases if the number of quiet periods is increased and the average channel utilization decreases.

To conclude this analysis, it is useful consider the case in which for each mini slot we collect $N$ samples, instead of $N/M$. In this case, as the number of mini slots increases, the probability of false alarm $P_{f,OR}$ decreases, as show in the next figure.

![Figure 81: Multi Slot Sensing - Probability of False versus L single slot - Logic OR (L for each mini slots equal to L)](image)

Intuitively, we would expect that the channel utilization increases when the number of quiet periods increases. Instead, as shown in the following figure, we obtain the higher channel utilization when the number of quiet periods is equal to 1. In fact, we must consider that in this case the total sensing time is proportional to the number of quiet periods. In fact, the optimum sensing time for which we obtain the maximum average channel utilization decreases if the number of quiet periods is increased. On the other hand, the maximum value of the average channel utilization decrease when the number of mini-slots is increased.
5.3.1.2 Logic AND Fusion Rule

Using equations (22), (41) and (42), we analyse the Average Channel Utilization as function of the sensing time $L$ when Logic AND fusion rule is adopted. The use of multi slot spectrum sensing with Logic AND Fusion Rule doesn’t increase the performance in term of average channel utilization. Again, this is due to the behaviour of the Probability of False Alarm $P_{f, AND}$, that is plotted in the next figure. As we can see, the probability of false alarm decreases if the sensing time $L$ is increased. Since the probability of false alarm increases if the number of quiet periods is increased, the average channel utilization decrease. In fact, in the CPUP scenario, it depend only on the $P_{f, AND}$ and $L$. Hence, for a fixed sensing time, the decrease of the channel utilization is due to the increase of the probability of false alarm.
Figure 83: Multi Slot Sensing - Average Channel Utilization versus Sensing Time (Logic AND Fusion Rule – Pidle=0.85)

Figure 84: Multi Slot Sensing - Probability of False Alarm versus Sensing Time (Logic AND Fusion Rule – Pidle=0.85)

As done in the case of Logic OR fusion Rule, we consider the following plot in which the probabilities of False Alarm, of the final decision and the single slot, and the Probability of Detection of the single slot are shown.
As we can see for a fixed probability of detection $P_{d,\text{AND}}$, the probabilities of false alarm and detection of single slot, $P_{f,0}$ and $P_{d,0}$, respectively, increase if the number of quiet periods is decreased. Moreover, like in the case of OR fusion rule, the probability of false alarm $P_{f,\text{AND}}$ increases if the number of quiet periods is increased, and the average channel utilization decreases.

![Graph showing probabilities of false alarm and detection](image)

**Figure 85: Multi Slot Sensing - Probabilities of False Alarm and Probability of Detection (Single Slot) versus M (Logic AND Fusion Rule – $P_d=0.9$, $P_{idle}=0.85$, $L=10$ ms)**

To conclude this analysis, it is useful consider the case in which for each mini slot we collect $N$ samples, instead of $N/M$. 
In this case, as the number of mini slots increases, the probability of false alarm $P_{f,\text{AND}}$ decreases, as shown in the previous figure.

Intuitively, we would expect that the channel utilization increases when the number of quiet periods increases. Instead, as shown in the following figure, we obtain the higher channel utilization when the number of quiet periods is equal to 1. In fact, we must consider that in this case the total sensing time is proportional to the number of quiet periods.
Figure 87: Multi Slot Sensing – Average Channel Utilization versus L single slot – Logic AND
(Number of sample for each mini slots equal to N)
5.3.2 Decision Fusion under CSUSU Scenario

In this subsection, considering both the Logic OR and Logic AND fusion rules, we analyse the performance of Decision Fusion in multi slot spectrum sensing under the Constant Secondary User Spectrum Utilization Scenario.

As done in the case of single slot, we use a target Probability of Detection of the final decision equal to 0.9, and, according with the DFS requirements, we consider the incumbent detection threshold of the DTV signal ($P_s = -116 \text{ dBm}$) and a noise level equal to $P_n = -95.2 \text{ dBm}$ (bandwidth equal to 6 MHz and $NF = 11 dB$).

5.3.2.1 Logic OR Fusion Rule

Using equations (22), (34) and (35), we analysis the Average Channel Utilization as function of the sensing time $L$ when Logic OR fusion rule is adopted.

![Figure 88: Multi Slot Sensing - Average Channel Utilization versus Sensing Time (Logic OR Fusion Rule – Pidle=0.85)](image)
As we can see in the previous figure, the use of multi slot spectrum sensing with Logic OR Fusion Rule increases the performance in term of channel utilization. Looking at the pictures above we are led to conclude that the use of multi-slot spectrum sensing with OR fusion rule leads to a performance’s increase in terms of Average Channel Utilization. In fact, as done in the case of single slot, the maximum value of Average Channel Utilization in the CSUSU scenario is obtained considering the time needed to achieve a sensing probability of detection equal to 0.9. For this reason, in the following figure we plot the probability of detection as a function of sensing time and, then, we get the value of \( L \) with which we achieve the minimum value of \( P_{d,OR} \) defined by the standard.

![Figure 89: Multi Slot Sensing – Probability of Detection versus Sensing Time – CSUSU scenario (Logic OR Fusion Rule – \( P_{idle}=0.85 \))](image)

As we can see, for a fixed \( P_{d,OR} \), the sensing time increases when the number of quiet periods increases. Using these values, we can compute the Average Channel Utilization, as shown in the next figure.
As shown the Average Channel Utilization decreases if the number of quiet periods increases. Hence, the seeming increase of the Channel utilization is due to the increase of the Probability of MisDetection, \( P_{md,OR} = 1 - P_{d,OR} \) and we can say that, again, the multi slot spectrum sensing using OR fusion rule doesn’t lead up to increase of the average channel utilization.

To conclude this analysis, it is useful consider the case in which for each mini slot we collect \( N \) samples, instead of \( N/M \).
In this case, as the number of mini slots increases, the probability of detection $P_{d,OR}$ increases, as shown in the previous figure.

Intuitively, we would expect that the channel utilization increases when the number of quiet periods increases because we obtain the target probability of detection for a smaller $L$. Instead, as shown in the following figure, we obtain the higher channel utilization when the number of quiet periods is equal to 1. In fact, we must consider that in this case the total sensing time is proportional to the number of quiet periods.
5.3.2.2 Logic AND Fusion Rule

Using equations (34) and (35), we plot the Average Channel Utilization as function of the sensing time $L$ when Logic AND fusion rule is adopted. As we can see in the next figure, the use of multi slot spectrum sensing with Logic AND Fusion Rule increases the performance in term of channel utilization. In the follow of this subsection we demonstrate that this increase of the channel utilization is due to the increase of the probability of misdetection, like when OR fusion rule is adopted. As said in the case of multi slot using OR fusion rule, the maximum value of Average Channel Utilization in the CSUSU scenario is obtained considering the time needed to achieve a sensing probability of detection equal to 0.9. For this reason, in the following figure we plot the probability of detection as a function of sensing time and, then, we get the value of $L$ with which we achieve the minimum value of $P_{d,OR}$ defined by the standard.
As we can see, for a fixed $P_{d, AND}$, the sensing time increases when the number of quiet periods increases.

**Figure 93: Multi Slot Sensing - Average Channel Utilization versus Sensing Time (Logic AND Fusion Rule – $P_{idle}=0.85$)**

**Figure 94: Multi Slot Sensing – Probability of Detection versus Sensing Time – CSUSU scenario (Logic AND Fusion Rule – $P_{idle}=0.85$)**
Using these value, we can compute the Average Channel Utilization, as shown in the next figure.

![Graph showing Average Channel Utilization](image)

**Figure 95: Multi Slot Sensing – Value of Average Channel Utilization - CSUSU scenario (Logic AND Fusion Rule – P_idle=0.85)**

As shown the Average Channel Utilization decreases if the number of quiet periods increases. Hence, the seeming increase of the Channel utilization is due to the increase of the Probability of MisDetection, $P_{md,AND} = 1 - P_{d,AND}$ and we can say that, again, the multi slot spectrum sensing using AND fusion rule doesn’t lead up to increase of the average channel utilization.

To conclude this analysis, it is useful consider the case in which for each mini slot we collect $N$ samples, instead of $N/M$. 
Figure 96: Multi Slot Sensing - Probability of False versus L single slot- Logic AND (Number of sample for each mini slots equal to N)

In this case, as the number of mini slots increases, the probability of detection \( P_{d,\text{AND}} \) increases, as show in the previous figure.

Intuitively, we would expect that the channel utilization increases when the number of quiet periods increases because we obtain the target probability of detection for a smaller \( L \). Instead, as shown in the following figure, we obtain the higher channel utilization when the number of quiet periods is equal to 1. In fact, we must consider that in this case the total sensing time is proportional to the number of quiet periods.
Figure 97: Multi Slot Sensing – Average Channel Utilization versus L single slot– Logic AND
(Number of sample for each mini slots equal to N)
REFERENCES


Chapter 6

COOPERATIVE SPECTRUM SENSING USING SINGLE-STAGE SENSING

In order to improve the performance of the spectrum sensing, the Secondary Users can cooperate to detect the presence of the Primary User. For this reason, in this section we extend the system model used in the Local Single Stage Sensing to the case of Cooperative Spectrum Sensing, i.e., a Cognitive Radio Network in which secondary users cooperate.

The decision topology used for cooperative detection is a parallel network with a fusion center. This topology consists of $N \geq 2$ local detectors all observing the same phenomenon. The local detectors transmit their measurement statistics to a fusion center which makes a global decision.

In this section we consider Hard Decision Fusion which means that each secondary user makes a local decision about the presence of primary user and then sends the binary decision to the fusion center for decision fusion. We consider performance’s analysis in the case of secondary users grouped in clusters, hence all with the same average Signal-to-Noise Ratio.

In this chapter we describe and analyse the performance in Cooperative Spectrum Sensing using Single Stage Sensing. In particular, according with the results of the previous chapter, we consider only Single Slot Sensing transmission mode.
6.1 Overview on Cooperative Spectrum Sensing

Hard Decision Fusion means that each user observes, in the case of energy detection, the signal energy in a given spectrum band, compares it to a threshold and makes a decision on the presence of a primary user according to the observation. Then, each cooperative node shares its decision with other radios using zero or one to inform whether they observe a free channel or an occupied channel, respectively. In particular, if the individual decision of the generic user $i$-th is equal to $H_0$, the user sends to the Base Station a flag equal to 0, while if the individual decision is equal to $H_1$, it sends to the base station a flag equal to 1. When the base station receives a flag, it made a final decision according with a fusion rule. In particular, each secondary user sense the environments for a time equal to $L$ and then makes the final decision at the end of the $CDT$ interval. The base station, or the master secondary user, collects the single decision and fuses it to make the final decision.

![Figure 98: Cooperative Single Slot Spectrum Sensing](image)

6.1.1 Maximum Number of Cooperative Users

The Draft PHY/MAC Specification for IEEE 802.22 WRANs provides a Multiple Access based on Orthogonal Frequency Division Multiple Access (OFDMA) according with the system parameters shown in the next table.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specification</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency range</td>
<td>54–862 MHz</td>
<td></td>
</tr>
<tr>
<td>Service coverage</td>
<td>Typical range 33 km</td>
<td></td>
</tr>
<tr>
<td>Bandwidth</td>
<td>Mandatory: 6, 7, 8 MHz</td>
<td>Optional fractional use of TV channel and channel bonding up to 3 contiguous TV channels. Channel aggregation of discontinuous channels.</td>
</tr>
<tr>
<td>Data rate</td>
<td>Maximum: 72.6 Mbps</td>
<td>Maximum of 23 Mbps for 6 MHz</td>
</tr>
<tr>
<td></td>
<td>Minimum: 4.8 Mbps</td>
<td></td>
</tr>
<tr>
<td>Spectral Efficiency</td>
<td>Maximum: 4.03 bits/s/Hz</td>
<td>Single TV channel BW of 6 MHz</td>
</tr>
<tr>
<td></td>
<td>Minimum: 0.81 bits/s/Hz</td>
<td></td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK, 16QAM, 64QAM</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mandatory</td>
<td></td>
</tr>
<tr>
<td>Transmit power</td>
<td>Default 4W EIRP</td>
<td></td>
</tr>
<tr>
<td>Multiple Access</td>
<td>Adaptive OFDMA</td>
<td>Partial bandwidth allocation</td>
</tr>
<tr>
<td>FFT Mode</td>
<td>2K mandatory</td>
<td>1K / 4K optional, 2K / 4K / 6K for channel bonding</td>
</tr>
<tr>
<td>Cyclic Prefix Mode</td>
<td>1/4, 1/8, 1/16, 1/32</td>
<td></td>
</tr>
<tr>
<td>Duplex</td>
<td>TDD mandatory</td>
<td>FDD supported</td>
</tr>
<tr>
<td>Network topology</td>
<td>Point-to-Multipoint Network</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Physical Standard Specification for IEEE 802.22

As we can see in the third column of Table 1, the multiple access based on adaptive OFDMA provides a partial bandwidth allocation. This means that the bandwidth $B_i$ available for the $i$th secondary user, $i=1,2,...,I$, is equal to

$$B_i = \frac{B}{N},$$  \hspace{1cm} (1) \hspace{1cm}

where $B$ is the total bandwidth. According to the partial bandwidth allocation, let $\eta$ be the Spectral Efficiency, defined as the ratio between the data rate and the bandwidth,
\[ \eta = \frac{R}{B} \text{ bit/(s/Hz)}, \]  

the maximum data rate available for each cooperative user is given by

\[ R_{\text{max},i} = \eta B_i \text{ bit/s} \]  

The maximum data rate using a single TV channel (6 MHz) depend on particular modulation used [1]. In particular, IEEE 802.22 defines 12 combinations of three modulations (quaternary phase shift keying [QPSK], 16-quadrature amplitude modulation [QAM], 64-QAM) and four coding rates (1/2, 2/3, 3/4, 5/6) for data communications that can be flexibly chosen among to achieve various trade-offs of data rate and robustness, depending on the channel and interference conditions. As shown in the next table, a total of 14 transmission modes are supported in IEEE 802.22.

<table>
<thead>
<tr>
<th>PHY Mode</th>
<th>Modulation</th>
<th>Coding Rate</th>
<th>Peak data rate in 6 MHz (Mb/s)</th>
<th>Spectral Efficiency (BW = 6 MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BPSK</td>
<td>Uncoded</td>
<td>4.54</td>
<td>0.76</td>
</tr>
<tr>
<td>2</td>
<td>QPSK</td>
<td>1/2 and repeat: 3</td>
<td>1.51</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>QPSK</td>
<td>1/2</td>
<td>4.54</td>
<td>0.76</td>
</tr>
<tr>
<td>4</td>
<td>QPSK</td>
<td>2/3</td>
<td>6.05</td>
<td>1.01</td>
</tr>
<tr>
<td>5</td>
<td>QPSK</td>
<td>3/4</td>
<td>6.81</td>
<td>1.13</td>
</tr>
<tr>
<td>6</td>
<td>QPSK</td>
<td>5/6</td>
<td>7.56</td>
<td>1.26</td>
</tr>
<tr>
<td>7</td>
<td>16-QAM</td>
<td>1/2</td>
<td>9.08</td>
<td>1.51</td>
</tr>
<tr>
<td>8</td>
<td>16-QAM</td>
<td>2/3</td>
<td>12.10</td>
<td>2.02</td>
</tr>
<tr>
<td>9</td>
<td>16-QAM</td>
<td>3/4</td>
<td>13.61</td>
<td>2.27</td>
</tr>
<tr>
<td>10</td>
<td>16-QAM</td>
<td>5/6</td>
<td>15.13</td>
<td>2.52</td>
</tr>
<tr>
<td>11</td>
<td>64-QAM</td>
<td>1/2</td>
<td>13.61</td>
<td>2.27</td>
</tr>
<tr>
<td>12</td>
<td>64-QAM</td>
<td>2/3</td>
<td>18.15</td>
<td>3.03</td>
</tr>
<tr>
<td>13</td>
<td>64-QAM</td>
<td>3/4</td>
<td>20.42</td>
<td>3.40</td>
</tr>
<tr>
<td>14</td>
<td>64-QAM</td>
<td>5/6</td>
<td>22.69</td>
<td>3.78</td>
</tr>
</tbody>
</table>

Table 4: PHY mode in IEEE 802.22
As we have seen, the maximum data rate is equal to 22.69 Mbit/s, to which correspond a Spectral Efficiency equal to 3.78 bit/s/Hz. Suppose that we allocate the same bandwidth for each user, using equations (1) and (2) and the result in (3), the maximum number of users that can share a single TV channel is given by

$$N = \left\lfloor \frac{\eta B}{R_{\min}} \right\rfloor,$$

(4)

where \(R_{\min}\) is the minimum data rate defined in the requirements of the IEEE 802.22 standard draft for a single user. The data rate constraints depend on whether the data is transmitted in the downlink or in the uplink. In the downlink (BS to CPE) the required minimum peak data rate must be at least equal to 1.5 Mbit/s per user, while in the uplink (CPE to BS) 384 Kbit/s per user, but, thanks to the adaptive boundary between upstream and downstream used in 802.22 MAC frames, the constraint per user rates can be combined in a single constraint, e.g., ~ 1.9 Mbit/s.

Hence, in order to obtain the minimum data rate per secondary user, the maximum theoretical number of users sharing a single TV channel is given by

$$N_{\text{max}} = \left\lfloor \frac{\eta B}{R_{\min}} \right\rfloor = \left\lfloor \frac{3.78 \text{ (bit/((s)Hz))} \times 6\text{ (MHz)}}{1.9 \text{ (Mbit/s)}} \right\rfloor = 11.$$

(5)

We say that the result in (5) is theoretical because, till this moment, we assumed that all time is dedicated for data transmission. In fact, since each transmission period is followed by a quiet period, using equation (3), the Average Throughput for a single user is given by

$$R_i = \rho R_{\text{max},i} = \rho \eta B_i.$$

(6)

where, since the IEEE 802.22 standard requires that all user to synchronize their quiet periods (to ensure the effective use of quiet period to improve sensing performance), \(\rho\) is average channel utilization of the cooperative users, equal for
each secondary user. Hence, in the cooperative spectrum sensing the sensing period must be chosen according to the number of cooperative users and the modulation used in order to satisfy the constraint about the minimum data rate $R_{\text{min}} = 1.9 \text{Mbits/s}$. 

6.1.2 Cooperative Sensing Accuracy

The extension of the sensing accuracy from local to cooperative spectrum sensing depends only on the number of cooperating users $I$ and the particular fusion rule adopted. Assuming that the decision of each user is independent from the decisions of the other users, $P_d^{(i)} = \ldots = P_d^{(i)} = P_d$ and $P_f^{(i)} = \ldots = P_f^{(i)} = P_f$ represent the probabilities of detection and false alarm of the single user, respectively, where $P_d$ and $P_f$ are given by equations (6) and (7) in Chapter 5. In this subsection we analyse the impact of the cooperative model on the performance of spectrum sensing using different fusion rules. In particular, the Joint Probabilities of detection, $Q_d$, and false alarm, $Q_f$, depend on the fusion rule and are given by

- **“LOGIC OR” fusion rule**: the decision is made that a primary user is present if one of the cooperating radios detects a primary user. In case all secondary users have the same individual $P_d$ and $P_f$, the joint probabilities of detection $Q_{d,\text{OR}}$ and false alarm $Q_{f,\text{OR}}$ can therefore be given as

$$Q_{d,\text{OR}} = 1 - (1 - P_d)^I$$  \hspace{1cm} (7) \\
$$Q_{f,\text{OR}} = 1 - (1 - P_f)^I$$  \hspace{1cm} (8)
• “LOGIC AND” fusion rule: the decision that a primary user is present is made only if all cooperating users detect the presence of primary user. In case all secondary users have the same individual $P_d$ and $P_f$, joint probabilities of detection $Q_{d,AND}$ and false alarm $Q_{f,AND}$ for $M$ cooperating users using AND-rule can be calculated as

$$Q_{d,AND} = P_d^I$$  (9)

$$Q_{f,AND} = P_f^I$$  (10)

### 6.1.3 Cooperative Channel Utilization

In the previous subsection we have seen that the sensing accuracy of Cooperative Spectrum Sensing can be characterized using equations (1) - (6). As seen in the Local Spectrum Sensing, the Average Channel Utilization $\rho$ depend on the probabilities that the WRAN decide for a idle or busy channel. In Cooperative Spectrum Sensing, these two probabilities can be express as follow

$$P_s = (1 - Q_f)P_{idle} + (1 - Q_d)P_{busy}$$  (11)

$$P_e = Q_f P_{idle} + Q_d P_{busy}$$  (12)

respectively, where and $Q_d$ and $Q_f$ depend on the particular fusion rule. Hence, using equations (11) and (12), the channel utilization of Cooperative Spectrum Sensing using Single Stage Sensing is given by

$$\rho = \left\{ (1 - Q_f)P_{idle} + (1 - Q_d)P_{busy}\right\} \left(\frac{T-L}{T}\right) + \left\{ Q_f P_{idle} + Q_d P_{busy}\right\} \left(\frac{T-L}{T_{tot}}\right)$$  (15)
where \( T_{tot} = T + T_{Move} + T_{Setup} \). The Channel Move Time and Channel Setup Time are independent on the number of cooperative users, but this is not true for Average Channel-Search Time.

6.1.3.1 Cooperative Average Channel-Search Time

In this subsection we analyse the impact of the cooperative scheme on the Average Channel-Search Time \( \bar{T}_{search} \). Using the results in chapter 5 and considering a conservative operation mode in which the channel is declared busy if only one cooperative user detects the primary user (Logic OR fusion rule), the Average Channel-search time can be express as follow:

\[
\bar{T}_{search} = \frac{T^s}{(1-Q_f)P_{idle}}. \tag{13}
\]

Hence, the optimum Sensing Time for each channel can be formulate as follow:

\[
\hat{T}^s = \arg \min_{T'} \frac{T^s}{(1-Q_f)P_{idle}} \quad T^s \geq T^s_{min} \quad Q_f = 1 - (1-P_f)^I \quad P_d = 1 - \sqrt[4]{1-Q_d} \quad P_f = Q \left( Q^{-1}(P_d) \sqrt{1 + 2SNR} + \sqrt[2]{\frac{T_s \cdot B \cdot SNR^2}{2}} \right) \tag{14}
\]

where \( T^s_{min} \) is the minimum sensing time to obtain the maximum value of the probability of false alarm. It can be compute as follow.
\[ T_{\text{min}}^s = \frac{2}{B} \left( \frac{Q^{-1}(P_f) - Q^{-1}(P_d)\sqrt{1 + 2\text{SNR}}}{\text{SNR}} \right), \]  

(15)

The trade off between the channel-search time and the quality of sensing can be observed plotting the average channel-search time as function of the sensing, as shown in the next figure, in which we consider a joint probability of detection equal to 0.99, a SNR equal to -20.8 dB, a bandwidth equal to 6 MHz and a $P_{idle} = 0.85$.

![Figure 99: Cooperative Single Slot - Average Channel-Search Time VS Sensing Time ($P_d=0.99$)](image)

As expected, when the number of cooperative users increases the $\bar{T}_{\text{search}}$ is decreased. In the next table the values of Average Channel-Search Time for different number of users are shown.
Table 5: Cooperative Single Slot - Optimum values of the Average channel-Search Time

<table>
<thead>
<tr>
<th></th>
<th>1 user</th>
<th>2 users</th>
<th>3 users</th>
<th>4 users</th>
<th>5 users</th>
<th>6 users</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{T}_{\text{search}}$</td>
<td>60.02 ms</td>
<td>37.54 ms</td>
<td>28.11 ms</td>
<td>23.15 ms</td>
<td>20.05 ms</td>
<td>17.90 ms</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>7 users</th>
<th>8 users</th>
<th>9 users</th>
<th>10 users</th>
<th>11 users</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{T}_{\text{search}}$</td>
<td>16.31 ms</td>
<td>15.08 ms</td>
<td>14.10 ms</td>
<td>13.29 ms</td>
<td>12.62 ms</td>
</tr>
</tbody>
</table>

In the next subsection, we’ll use these values to compute the average channel utilization in (15).

### 6.2 Cooperative Constant Primary User Protection Scenario

In Cooperative Single Stage Spectrum Sensing the CPUP scenario is realized by fixing the joint probability of detection at the required level $\bar{Q}_d$ and minimizing the joint probability of false alarm $Q_f$ as much as possible. In this scenario the average channel utilization can be express as follow

$$\rho = \left(1 - Q_f\right)P_{\text{idle}} + \left(1 - \bar{Q}_d\right)P_{\text{busy}} \left[\frac{T - L}{T}\right] + \left[Q_f P_{\text{idle}} + \bar{Q}_d P_{\text{busy}}\right] \left[\frac{T - L}{T_{\text{tot}}}\right]$$

where the joint probability of false alarm can be computed from the probability of false alarm of each user, the number of cooperative users and according with the fusion rule adopted. In other words, from the joint probability of detection, the procedure to compute $Q_f$ is the following:

1. Compute the probability of detection for each user, $\bar{P}_d$, from the number of cooperative users and according with the fusion rule adopted;
2. Compute the probability of false alarm for each user, \( P_f \), using the following equation

\[
P_f = Q\left(Q^{-1}(P_d)\sqrt{1+2\text{SNR}} + \sqrt{\frac{L \cdot B \cdot \text{SNR}^2}{2}}\right);
\]

3. Compute the joint probability of false alarm, \( Q_f \), from the number of cooperative users and according with the fusion rule adopted.

As known, the use of cooperative model increases the performance of spectrum sensing. In this subsection, using the fusion rules defined above, we investigate this issue when CPUP is adopted.

To do this, according with the DFS requirements, we consider the incumbent detection threshold of the DTV signal (\( P_s = -116 \text{ dBm} \)) and a noise level equal to \( P_n = -95.2 \text{ dBm} \) (bandwidth equal to 6 MHz and \( NF = 11 \text{ dB} \)).

### 6.2.1 Logic OR Fusion Rule under CPUP Scenario

Using equations (1) and (2), we analysis the Average Channel Utilization as function of the sensing time \( L \) when Logic OR fusion rule is adopted. In particular, the procedure defined above can be express as follow:

1. Compute the probability of detection for each user, \( \overline{P}_d \), from the number of cooperative

\[
\overline{P}_d = 1 - \sqrt{1 - Q_{d,OR}};
\]
2. Compute the probability of false alarm for each user, $P_f$, using equation (18);

3. Compute the joint probability of false alarm, $Q_f$, from the number of cooperative users

$$Q_{f,OR} = 1 - (1 - P_f)^I.$$ (19)

The behaviour of the average channel utilization is shown in the next figure.

![Average Channel Utilization versus Sensing Time](image)

**Figure 100: Cooperative Spectrum Sensing CPUP - Average Channel Utilization versus Sensing Time (Logic OR Fusion Rule – $P_{idle}=0.85$)**

As we can see in the next figure, when Logic OR Fusion Rule is used the average channel utilization increases. This increased performance is due to the behaviour of the Joint Probability of False Alarm $Q_{f,OR}$, that is plot in the next figure.
As we can see, the joint probability of false alarm decreases if the sensing time $L$ is increased. Since the probability of false alarm increases if the number of cooperative users is increased, the average channel utilization increases.

### 6.2.2 Logic AND Fusion Rule under CPUP Scenario

Using equations (3) and (4), we analysis the Average Channel Utilization as function of the sensing time $L$ when Logic AND fusion rule is adopted. In particular, the procedure defined above can be express as follow:

1. Compute the probability of detection for each user, $\overline{P}_d$, from the number of cooperative

$$\overline{P}_d = \sqrt{Q_{d,AND}}; \quad (20)$$
2. Compute the probability of false alarm for each user, $P_f$, using equation (20);

3. Compute the joint probability of false alarm, $Q_f$, from the number of cooperative users

$$Q_{f,OR} = P_f^I. \quad (21)$$

The behaviour of the average channel utilization is shown in the next figure.

![Graph showing average channel utilization versus sensing time for 1, 2, 5, and 11 users]

**Figure 102: Cooperative Spectrum Sensing CPUP - Average Channel Utilization versus Sensing Time (Logic AND Fusion Rule – $P_{idle}=0.85$)**

As we can see in the next figure, when Logic AND Fusion Rule is used the average channel utilization increases. This increase of performance is due to the behaviour of the Joint Probability of False Alarm $Q_{f,AND}$, that is plotted in the next figure.
As we can see, the joint probability of false alarm decreases if the sensing time $L$ is increased. Since the probability of false alarm increases if the number of cooperative users is increased, the average channel utilization increases.

### 6.2.3 Comparison between Logic OR and Logic AND

In this subsection we determine the optimum fusion rule that maximize the average channel utilization under CPUP scenario. To do this, we consider the following figure.
Figure 104: Cooperative Spectrum Sensing – CPUP - Comparison between Logic OR and Logic AND fusion rules

The previous figure shows that the benefits of cooperation can be achieved with a relative small number of cooperating users. In particular, these benefits are higher when Logic AND fusion rule is adopted. On the other hand, as shown in the next figure, to achieve the target joint probability of detection, each secondary user must guaranteed a probability of detection higher than the target.

Figure 105: Probabilities of Detection (Qd and Pd) vs Number of Cooperating Users (AND rule)
On the contrary, when Logic OR fusion rule is adopted, the probability of detection of each user decreases if the number of cooperative users is increased, as shown in the next figure.

![Graph showing Joint Probability and Probability of Detection vs Number of Cooperating Users](image)

**Figure 106: Probabilities of Detection (Qd and Pd) vs Number of Cooperating Users (OR rule)**

Hence, we can conclude that Logic AND fusion rule is the optimum fusion rule that maximize the average channel utilization, but we need more sophisticated sensor ($P_d$ must increases to achieve the target $Q_d$). On the other hand, when Logic OR fusion rule, we obtain a smaller performance improvement, but we need less sophisticated sensors.
6.3 Cooperative Constant Secondary User Spectrum Utilization Scenario

In Cooperative Single Stage Spectrum Sensing the CSUSU scenario is realized by fixing the joint probability of false alarm at the required level $Q_f$. The sensing time is chosen in order to obtain the minimum joint probability of detection $Q_d$ defined in the requirements about sensing accuracy of IEEE 802.22 standard. In this scenario the average channel utilization can be expressed as follows:

$$
\rho = \left[ (1 - Q_f) P_{idle} + (1 - Q_d) P_{busy} \right] \left( \frac{T - L}{T} \right) + \left[ Q_f P_{idle} + Q_d P_{busy} \right] \left( \frac{T - L}{T_{tot}} \right) 
$$  \hspace{1cm} (22)

where the joint probability of detection can be computed from the probability of false alarm of each user, the number of cooperative users and according with the fusion rule adopted. In other words, from the joint probability of false alarm, the procedure to compute $Q_d$ is the following:

1. Compute the probability of false alarm for each user, $P_f$, from the number of cooperative users and according with the fusion rule adopted;
2. Compute the probability of detection for each user, $P_d$, using the following equation

$$
P_d = Q \left[ Q^{-1}(P_f) - \frac{L \cdot B \cdot SNR^2}{\sqrt{1 + 2SNR}} \right] ; 
$$ \hspace{1cm} (23)

3. Compute the joint probability of detection, $Q_f$, from the number of cooperative users and according with the fusion rule adopted.
As known, the use of cooperative model increases the performance of spectrum sensing. In this subsection, using the fusion rules defined above, we investigate this issue when CSUSU is adopted.

To do this, according with the DFS requirements, we consider the incumbent detection threshold of the DTV signal \( P_s = -116 \text{ dBm} \) and a noise level equal to \( P_n = -95.2 \text{ dBm} \) (bandwidth equal to 6 MHz and \( NF = 11 \text{ dB} \)).

### 6.3.1 Logic OR Fusion Rule under CSUSU Scenario

Using equations (1) and (2), we analyse the Average Channel Utilization as function of the sensing time \( L \) when Logic OR fusion rule is adopted. In particular, the procedure defined above can be express as follow:

1. Compute the probability of false alarm for each user, \( \overline{P}_f \), from the number of cooperative

   \[
   \overline{P}_f = 1 - \sqrt{1 - Q_{f,\text{OR}}} ;
   \]

2. Compute the probability of detection for each user, \( P_d \), using equation (23);

3. Compute the joint probability of detection, \( Q_d \), from the number of cooperative users

   \[
   Q_{d,\text{OR}} = 1 - (1 - P_d)^L.
   \]

The behaviour of the Joint Probability of Detection as function of the sensing time is shown in the next figure.
As we can see in the next figure, it's possible to increase the probability of detection $Q_{d,OR}$ with an increased cooperating users’ number.

Figure 107: Cooperative Spectrum Sensing CSUSU – Average Channel Utilization versus Sensing Time (Logic OR Fusion Rule)

Figure 108: Cooperative Spectrum Sensing CSUSU - Joint Probability of Detection versus Sensing Time (Logic OR Fusion Rule – $P_{idle}=0.85$)
Using the sensing times that allowed to achieve a joint probability of detection equal to 0.9, we can compute the average channel utilization, as shown in the next figure.

![Figure 109: Cooperative Spectrum Sensing CSUSU– Values of Average Channel Utilization versus Sensing Time (Logic OR Fusion Rule)](image)

As expected, the Average Channel Utilization increases as the number of cooperating users increases.

### 6.3.2 Logic AND Fusion Rule under CSUSU Scenario

Using equations (3) and (4), we analysis the Average Channel Utilization as function of the sensing time $L$ when Logic AND fusion rule is adopted. In particular, the procedure defined above can be express as follow:

1. Compute the probability of false alarm for each user, $\overline{P_f}$, from the number of cooperative
The behaviour of the Joint Probability of Detection as function of the sensing time is shown in the next figure.

![Figure 110: Cooperative Spectrum Sensing CSUSU – Average Channel Utilization versus Sensing Time (Logic AND Fusion Rule)](image_url)

Again, as we can see in the next figure, for a fixed sensing time, the joint probability of detection increases if the number of cooperative user is increased.
Using the sensing times that allowed to achieve a joint probability of detection equal to 0.9, we can compute the average channel utilization, as shown in the next figure.

Figure 112: Cooperative Spectrum Sensing CSUSU– Values of Average Channel Utilization versus Sensing Time (Logic AND Fusion Rule)
As expected, the Average Channel Utilization increases as the number of cooperating users increases.

### 6.3.3 Comparison between Logic OR and Logic AND

In this subsection we determine the optimum fusion rule that maximize the average channel utilization under CSUSU scenario. To do this, we consider the following figure.

![Graph](image)

**Figure 113: Cooperative Spectrum Sensing – CSUSU -Comparison between Logic OR and Logic AND fusion rules**

The previous figure shows that the benefits of cooperation can be achieved with a relative small number of cooperating users. In particular, these benefits are higher when Logic OR fusion rule is adopted.
REFERENCES

CONCLUSION

In this work we determined the optimum sensing period for spectrum sensing. As shown in the previous section, if we use mini slot spectrum sensing we don’t obtain a performance’s increase in term of average channel utilization. Hence, the optimum transmission mode is the single slot spectrum sensing, to which correspond a sensing period equal to 2 seconds. Moreover, using a CPUP scenario, we can set the optimum sensing time in order to maximize the average channel utilization, higher than single slot spectrum sensing under CSUSU scenario.

As we have seen in Chapter 6, the Cooperative Spectrum Sensing increases the average channel utilization for each user, hence, the data rate of the network increases. On the other hand, the data rate of each user decreases if the number of cooperative users increase. For this reason, after the selection of the sensing time, the base station must evaluate the average throughput of each secondary user in order to guarantee the minimum data rate defined by the standard requirements of IEEE 802.22. Moreover, since we considered a fixed values of the SNR, the base station should estimate the received SNR in order to obtain a optimum choice of the sensing time. This represents the cognitive capability of a sensing device, with which a cognitive radio can understand the context it finds itself in and autonomously configure itself in response to a set of goals. In this case the goal is the optimum selection of the sensing time in order to satisfy the requirements about the sensing accuracy and capacity. Hence, this issue can be formulated as an optimization problem and solved using numerical optimization.